**2013 Workshop on Matrices and Operators**
**Beijing Normal University, July 15-19, 2013**
**Abstract**

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**Name:** Tsuyoshi ANDO, (ando@es.hokudai.ac.jp)

**Affiliation:** Hokkaido University, Japan (Emeritus)

**Title:** Majorization relations involving partial traces

**Abstract:** Let $\mathrm{M}_k$ denote the space of $k \times k$ complex matrices.

Given $m, n \geq 2$ the tensor product space $\mathrm{M}_m \otimes \mathrm{M}_n$, the space of block-matrices $\mathrm{M}_m(\mathrm{M}_n)$ with entries in $\mathrm{M}_n$, and the space $\mathrm{M}_{mn}$ are canonically identified as

$$\mathrm{M}_m \otimes \mathrm{M}_n \ \ni \ [\alpha_{j,k}]_{j,k=1}^m \otimes B \ \simeq \ [\alpha_{j,k}B]_{j,k=1}^m \ \in \ \mathrm{M}_m(\mathrm{M}_n).$$

The order relation is induced by the cone of positive semi-definite matrices.

The imbedding linear map $\mathrm{M}_m \ni X \ \longmapsto X \otimes I_n \ \in \mathrm{M}_m \otimes \mathrm{M}_n$ is positive and its adjoint map $\varphi_1$ is called the *partial trace* (map) from $\mathrm{M}_m \otimes \mathrm{M}_n$ to $\mathrm{M}_m$. It is again a positive linear map. Correspondingly the partial trace $\varphi_2$ from $\mathrm{M}_m \otimes \mathrm{M}_n$ to $\mathrm{M}_n$ is defined. More explicitly they are written as

$$\varphi_1(\mathbf{S}) = [\mathrm{Tr}(S_{j,k})]_{j,k=1}^m \text{ and } \varphi_2(\mathbf{S}) = \sum_{j=1}^m S_{j,j} \quad \forall\, \mathbf{S} = [S_{j,k}]_{j,k=1}^m \in \mathrm{M}_m(\mathrm{M}_n).$$

The notions of partial traces are quite common in physics. A matrix $\mathbf{S} \geq 0$ with $\mathrm{Tr}(\mathbf{S}) = 1$ is usually called a *density matrix*, and $\varphi_1(\mathbf{S})$ and $\varphi_2(\mathbf{S})$ are called its *reduced density matrices*.

The notion of (eigenvalue) *majorization* between two positive semi-definite matrices of same order is extended to the case of $A, B \geq 0$ of different order as, with $l := \min\Big(\text{order of } A, \text{ order of } B\Big)$,

$$A \succ B \qquad \Longleftrightarrow \qquad \sum_{j=1}^k \lambda_j^{\downarrow}(A) \ \geq \ \sum_{j=1}^k \lambda_j^{\downarrow}(B) \quad (k = 1, 2 \ldots, l) \quad \text{and} \quad \mathrm{Tr}(A) = \mathrm{Tr}(B).$$

The most elementary but fundamental fact is that $AA^* \succ A^*A$ ($\succ AA^*$) for any rectangular matrix $A$.

Physicists discovered the following majorization relations under suitable conditions on $\mathbf{S} \geq 0$ :

$$\varphi_1(\mathbf{S}) \succ \mathbf{S} \quad \text{and} \quad \varphi_2(\mathbf{S}) \succ \mathbf{S}.$$

Among such conditions are the matrix inequalities:

$$\varphi_1(\mathbf{S}) \otimes I_n \geq \mathbf{S} \quad \text{and} \quad I_m \otimes \varphi_2(\mathbf{S}) \geq \mathbf{S}.$$

In this talk we present a little different approaches to those facts.

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**Name:** Richard A. Brualdi (brualdi@math.wisc.edu).

**Affiliation:** University of Wisconsin - Madison

**Title:** Alternating Sign Matrices and Their Completions

**Abstract:** An Alternating Sign Matrix (ASM) is a $(0, \pm 1)$-matrix such that, ignoring 0s, the $+1$s and $-1$s in each row and column alternate beginning and ending with a $+1$. All the row and column sums equal 1 in an ASM. After a discussion of the highlights concerning ASMs, we will discuss some recent work concerning ASM completions.

**Co-author(s):** Hwa-Kyung Kim

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**Name:** Chi-Tung Chang (d937208@oz.nthu.edu.tw)

**Affiliation:** Department of Applied Mathematics, Feng Chia University, Taiwan

**Title:** Equality of Higher-rank Numerical Ranges of Matrices

**Abstract:** Let $\Lambda_k(A)$ denote the rank-$k$ numerical range of an $n$-by-$n$ complex matrix $A$. We give a characterization for $\Lambda_{k_1}(A) = \Lambda_{k_2}(A)$, where $1 \leq k_1 \leq k_2 \leq n$, via the compressions and the principal submatrices of $A$. As an application, the matrix $A$ satisfying $W(A) = \Lambda_k(A)$, where $W(A)$ is the classical numerical range of $A$ and $1 \leq k \leq n$, is under consideration. We show that if $W(A) = \Lambda_k(A)$ for some $k > n/3$, then $A$ is unitarily similar to $\underbrace{B \oplus B \oplus \cdots \oplus B}_{3k-n \text{ copies}} \oplus C$, where $B$ is a 2-by-2 matrix, $C$ is a $(3n-6k)$-by-$(3n-6k)$ matrix and $W(A) = W(B) = W(C) = \Lambda_{n-2k}(C)$.

**Co-author(s):** Hwa-Long Gau and Kuo-Zhong Wang

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**Name:** Mao-Ting Chien (mtchien@scu.edu.tw)

**Affiliation:** Soochow University, Taiwan

**Title:** Reduction of $c$-numerical ranges

**Abstract:** The classical numerical range of a matrix has been extensively studied. One of generalizations of the classical numerical range is $c$-numerical range. We show that for any $n$-by-$n$ matrix $A$ and real $n$-tuple vector $c$, the $c$-numerical range of $A$ is attainable by the classical numerical range of a matrix $B$ of size at most $n!$. Constructions of the matrix $B$ for some matrices $A$ and real $n$-tuple $c$ are given.

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**Name:** Man-Duen Choi(choi@math.toronto.edu)

**Affiliation:** Mathematics Department, University of Toronto

**Title:** Square and Circle mystified

**Abstract:** The easy geometry of square and circle has appeared naturally in the theory of non-commutative harmonic analysis. Surprisingly, there remain simple questions (with unknown depth) of matrix theory.

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**Name:** Ajda Fošner

**Affiliation:** University of Primorska

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**Title:** Some results on preservers on tensor states

**Abstract:** We characterize linear operators $\phi$ on $mn \times mn$ Hermitian matrices such that $\phi(A \otimes B)$ and $A \otimes B$ have the same spectrum for any $m \times m$ Hermitian $A$ and $n \times n$ Hermitian $B$. Such a map has the form $A \otimes B \mapsto U(\varphi_1(A) \otimes \varphi_2(B))U^*$ for $mn \times mn$ Hermitian matrices in tensor form $A \otimes B$, where $U$ is a unitary matrix, and for $j \in \{1, 2\}$, $\varphi_j$ is the identity map $X \mapsto X$ or the transposition map $X \mapsto X^t$. The structure of linear maps leaving invariant the spectral radius of matrices in tensor form $A \otimes B$ is also obtained. The results are extended to multipartite systems.

**Co-authors:** Zejun Haung, Chi-Kwong Li, Nung-Sing Sze

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**Name:** Hwa-Long Gau (hlgau@math.ncu.edu.tw)

**Affiliation:** National Central University, Taiwan

**Title:** Zero-dilation index of a finite matrix

**Abstract:** For an $n$-by-$n$ complex matrix $A$, we define its zero-dilation index $d(A)$ as the largest size of a zero matrix which can be dilated to $A$. This is the same as the maximum $k$ ($\geq 1$) for which 0 is in the rank-$k$ numerical range of $A$. Using a result of Li and Sze, we show that if $d(A) > \lfloor 2n/3 \rfloor$, then, under unitary similarity, $A$ has the zero matrix of size $3d(A) - 2n$ as a direct summand. It complements the known fact that if $d(A) > \lfloor n/2 \rfloor$, then 0 is an eigenvalue of $A$. We then use it to give a complete characterization of $n$-by-$n$ matrices $A$ with $d(A) = n - 1$, namely, $A$ satisfies this condition if and only if it is unitarily similar to $B \oplus 0_{n-3}$, where $B$ is a 3-by-3 matrix whose numerical range $W(B)$ is an elliptic disc and whose eigenvalue other than the two foci of $\partial W(B)$ is 0. We also determine the value of $d(A)$ for any normal matrix and any weighted permutation matrix with zero diagonals $A$.

**Co-author(s):** Kuo-Zhong Wang, Pei Yuan Wu.

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**Name:** Zejun Huang (mathzejun@gmail.com)

**Affiliation:** The Hong Kong Polytechnic University

**Title:** Linear maps preserving the higher numerical ranges of tensor products of matrices

**Abstract:** For a positive integer $k$, let $M_k$ be the set of $k \times k$ complex matrices. Suppose $m, n \geq 2$ are positive integers. In this talk, we will present the characterization of linear maps $\phi$ on $M_{mn}$ leaving invariant the higher numerical ranges of matrices in tensor form $A \otimes B$ with $A \in M_m$ and $B \in M_n$.

**Co-authors:** Ajda Fošner, Chi-Kwong Li, Yiu-Tung Poon, and Nung-Sing Sze.

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**Name:** Chi-Kwong Li (ckli@math.wm.edu)

**Affiliation:** College of William and Mary

**Title:** Decomposition of unitary matrices

**Abstract:** We discuss problems on factorization of unitary matrices into matrices with prescribed zero and nonzero patterns to facilitate the implementation of quantum operations.

**Co-authors:** Diane Pelejo, Rebecca Roberts, Xiaoyan Yin.

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**Speaker:** Zhongshan Li (zli@gsu.edu)

**Affiliation:** Georgia State University and North University of China.

**Title:** Polytopes and nonnegative sign patterns

**Co-authors:** W. Fang, W. Gao, Y. Gao, F. Gong, G. Jing, and Y. Shao.

**Abstract:** A *sign pattern (matrix)* is a matrix whose entries are from the set $\{+, -, 0\}$. The *minimum rank* (respectively, *rational minimum rank*) of a sign pattern matrix $\mathcal{A}$ is the minimum of the ranks of the real (respectively, rational) matrices whose entries have signs equal to the corresponding entries of $\mathcal{A}$. A new direct connection between polytopes (and more generally, polyhedrons) and nonnegative matrices (or nonnegative sign patterns) is presented. Rational realizability or nonrealizability (of the combinatorial types) of polytopes has been an important theme in the study of polytopes. In particular, it is known that every 3-polytope is rationally realizable, but there are 4-polytopes (and $d$-polytopes for $d \geq 5$) that are not rationally realizable. Using these results, we establish that for every nonnegative sign pattern with minimum rank less than or equal to 4, the minimum rank can be achieved over the rationals; but for each integer $k \geq 5$, there are nonnegative sign patterns with minimum rank $k$ that do not permit rational realization of the minimum rank.

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**Name:** Ming-Huat Lim (limmh@um.edu.my)

**Affiliation:** University of Malaya, Malaysia

**Title:** Preservers of symmetric Boolean matrices of term rank two

**Abstract:** Let $\mathcal{B}$ denote the two element Boolean algebra and $S_n(\mathcal{B})$ be the semimodule of all $n$-square symmetric Boolean matrices. We characterize (i) subsemimodules of $S_n(\mathcal{B})$ whose nonzero members all have term rank less than 3 where $3 \leq n$, (ii) linear mappings from $S_n(\mathcal{B})$ and $S_m(\mathcal{B})$ that send distinct elements of term rank 2 to distinct elements of term rank 2 where $3 \leq n$, (iii) linear mappings from $S_n(\mathcal{B})$ to $S_m(\mathcal{B})$ that preserve elements of term rank 2 and also elements of term rank $k$ for some $3 \leq k \leq n$, and (iv) linear mappings from $S_n(\mathcal{B})$ to $S_m(\mathcal{B})$ that send distinct nonzero matrices of term rank less than three to distinct nonzero matrices of term rank less than three where $3 \leq n$. All the above results are proved in the more general setting of symmetric Boolean tensors of order two.

**Name:** Yongdo Lim (ylim@skku.edu)

**Affiliation:** Sungkyunkwan University

**Title:** Hadamard metrics on the cone of positive definite matrices

**Abstract:** We present a new class of (metric) geometric means of positive definite matrices varying over Hermitian unitary matrices. We show that each Hermitian unitary matrix induces a factorization of the cone $P_m$ of $m \times m$ positive definite Hermitian matrices into geodesically convex subsets and a Hadamard metric (NPC) structure on $P_m$. An explicit formula for the corresponding metric midpoint operation is presented in terms of the geometric and spectral geometric means and show that the resulting two-variable mean is different to the standard geometric mean.

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**Speaker:** Yiu-Tung Poon (ytpoon@iastate.edu)

**Affiliation:** Iowa State University, USA

**Title:** Quantum states with the same reduced states

**Abstract:** We discuss results and problems on quantum states of a composite system with prescribed reduced states

**Co-authors:** Chi-Kwong Li and Xuefeng Wang.

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**Name:** Wasin So (wasin.so@sjsu.edu)

**Affiliation:** Department of Mathematics, San Jose State University, San Jose, CA 95192, USA

**Title:** Inverse Graph Eigenvalue Problem

**Abstract:** The eigenvalues of a simple undirected graph are the eigenvalues of its adjacency matrix. There is a huge literature on the study of graph eigenvalues. We are intrested in the inverse problems: *Which numbers are the eigenvalues of a simple undirected graph?* In this talk, we will mention some old results from the literature, propose some new problems, and share some partial answers.

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**Name:** Raymond Nung-Sing Sze (raymond.sze@polyu.edu.hk)

**Affiliation:** Department of Applied Mathematics, The Hong Kong Polytechnic University, Hong Kong

**Title:** Entanglement transformation of quantum states using local operations

**Abstract:** A local operation (LO) in a bipartite system $M_{mn}$ is a trace preserving completely positive (TPCP) map of the form

$$A \mapsto \sum_{i=1}^{p} \sum_{j=1}^{q} (F_i \otimes G_j) A (F_i \otimes G_j)^*,$$

where $F_1, \ldots, F_p \in M_m$ and $G_1, \ldots, G_q \in M_n$ satisfy $\sum_{i=1}^{p} F_i^* F_i = I_m$ and $\sum_{i=1}^{q} G_j^* G_j = I_n$. Given two sets of pure states in a bipartite system, represented as sets of rank one density matrices $\{x_1 x_1^*, \ldots, x_k x_k^*\}$ and $\{y_1 y_1^*, \ldots, y_k y_k^*\}$, we give necessary and sufficient conditions on the existence of a local operation transforming between the two sets of states. Also efficient algorithms are provided to rule out its existence if a LO transformation is not possible.

**Co-author(s):** Hoi-Fung Chau, Chi-Hang Fred Fung, Chi-Kwong Li, and Edward Poon.

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**Name:** Bit-Shun Tam (bsm01@mail.tku.edu.tw)

**Affiliation:** Tamkang University, Taiwan

**Title:** On the signless Laplacian coefficients of unicyclic graphs

**Abstract:** Let $G$ be a graph of order $n$ and let $Q_G(x) = \sum_{i=0}^{n} (-1)^i p_i(G) x^{n-i}$ be the characteristic polynomial of the signless Laplacian of $G$. Let $E_{g,n}$ (respectively, $C_g(S_{n-g+1})$) denote the unicyclic graph of order $n$ obtained by a coalescence of a vertex in the cycle $C_g$ with an end vertex of the path $P_{n-g+1}$ (respectively, with the center of the star $S_{n-g+1}$). It is proved that for $k = 2, \ldots, n-1$, as $G$ varies over all unicyclic graphs of order $n$, depending on $k$ and $n$, the maximum value of $p_k(G)$ is attained at $G = C_n$ or $E_{3,n}$, and the minimum value is attained uniquely at $G = C_4(S_{n-3})$ or $C_3(S_{n-2})$. Except for the resolution of a conjecture on cubic polynomials, the uniqueness issue for the maximization problem is also settled.

**Co-authors:** Hong-Hai Li and Li Su (Jiangxi Normal University)

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**Name:** Tin-Yau, Tam (tamtiny@auburn.edu)

**Affiliation:** Department of Mathematics, Auburn University.

**Title:** Geometric means for positive definite matrices and beyond

**Abstract:** We discuss some results of Bhatia et al that involve geometric means $A \#_{1/2} B = A^{1/2} \left(A^{-1/2} B A^{-1/2}\right)^{1/2} A^{1/2}$ of two $n \times n$ positive definite matrices $A$ and $B$. The geometric means lies on the geodesic joining $A$ and $B$. We then consider their extensions in the context of symmetric space of the noncompact type. Some partial order relation is given.

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**Name:** Ngai-Ching Wong (wong@math.nsysu.edu.tw)

**Affiliation:** Department of Applied Mathematics, National Sun Yat-sen University, Kaohsiung 80424, Taiwan.

**Title:** Kaplansky Theorem for completely regular spaces

**Abstract:** Let $X, Y$ be real compact spaces or completely regular spaces consisting of $G_\delta$-points. Let $\phi$ be a linear bijective map from $C(X)$ (resp. $C^b(X)$) onto $C(Y)$ (resp. $C^b(Y)$). We show that if $\phi$ preserves non-vanishing functions, that is,

$$f(x) \neq 0, \forall\, x \in X, \quad \Longleftrightarrow \quad \phi(f)(y) \neq 0, \forall\, y \in Y,$$

then $\phi$ is a weighted composition operator

$$\phi(f) = \phi(1) \cdot f \circ \tau,$$

arising from a homeomorphism $\tau$ from $Y$ onto $X$. This result is applied also to other nice function spaces, e.g., uniformly or Lipschitz continuous functions on metric spaces.

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**Name:** Pei Yuan Wu (pywu@math.nctu.edu.tw)

**Affiliation:** National Chiaotung University, Taiwan.

**Title:** Numerical ranges of KMS matrices

**Abstract:** A KMS matrix, denoted by $J_n(a)$, is an $n$-by-$n$ upper triangular matrix whose diagonal entries are all 0 and whose $(i, j)$-entry for $i$ strictly less than $j$ equals the $(j - i)$th power of some scalar $a$. Such matrices arise from the work of Kac, Murdock and Szego from 1953, and hence the namesake. They are at the meeting ground of the classes of nilpotent matrices, Toeplitz matrices, nonnegative matrices, $S_n$-matrices and $S_n^{-1}$-matrices, and thus have many interesting properties concerning their numerical ranges. We will consider

(1) when its numerical range is a circular disc,

(2) when the boundary of its numerical range contains a line segment, and

(3) when the boundary of its numerical range intersects that of one of its principal submatrices.

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**Name:** Xingzhi Zhan (zhan@math.ecnu.edu.cn)

**Affiliation:** East China Normal University

**Title:** Extremal sparsity of the companion matrix of a polynomial

**Abstract:** We present a result on the extremal sparsity of the companion matrix of a polynomial, and explain the basic ideas in the proof.

**Co-author:** Chao Ma

**Name:** Fuzhen Zhang (zhang@nova.edu)

**Affiliation:** Nova Southeastern University, Ft. Lauderdale, Florida, USA

**Title:** Embedding, Tensor Product, and Generalized Matrix Functions

**Abstract:** We briefly review classic results on generalized matrix functions, propose a few new problems, and show new inequalities for positive semidefinite matrices and generalized matrix functions via an embedding approach and tensor product.