2015 Workshop on Matrices and Operators

Titles and Abstracts

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Affiliation: Hokkaido Univ. (Emeritus)

Title: Positive maps; linear and non-linear

Abstract: A linear or non-linear map from $\mathbb{M}_n$ to $\mathbb{M}_N$ is said to be positive if it transforms $\mathbb{M}_n^+$ to $\mathbb{M}_N^+$.

We show that on $\mathbb{M}_m(\mathbb{M}_n)$, the space of $m \times m$ matrices with entries in $\mathbb{M}_n$, the maps $\Omega_{m,n}$ and $\Pi_{m,n}$ defined for $\mathbf{S} = [S_{jk}]_{j,k}$ as

$$\Omega_{m,n}(\mathbf{S}) := \mathrm{Tr}(\mathbf{S}) \cdot \Delta(\mathbf{S}) - [\mathrm{Tr}(S_{jk})]_{j,k}^T \circ \mathbf{S} - \Delta(\mathbf{S}) \cdot \mathbf{J} \cdot \Delta(\mathbf{S}) + \Delta(\mathbf{S}^2)$$

and

$$\Pi_{m,n}(X) := \mathrm{Tr}(\mathbf{S}) \cdot \Delta(\mathbf{S}) - [\mathrm{Tr}(S_{jk})]_{j,k}^T \circ \mathbf{S} + \Delta(\mathbf{S}) \cdot \mathbf{J} \cdot \Delta(\mathbf{S}) - \Delta(\mathbf{S}^2)$$

are positive, where $\Delta(\mathbf{S}) := \mathrm{diag}(S_{11}, \ldots, S_{mm})$, $[..]^T$ transpose, $\circ$ the generalized Schur product, and $\mathbf{J} = [I_n]_{j,k}$ with all entries I_n.

Those positivities are closely connected with positivities of the tensor products $\Phi_m \otimes \Phi_n$ and $\Phi_m \otimes \Psi_n$ of a linear map Φ_m on $\mathbb{M}_m$ and linear maps Φ_n and Ψ_n on $\mathbb{M}_n$ where for each n

$$\Phi_n(X) := \mathrm{Tr}(X) \cdot I_n - X \quad \text{and} \quad \Psi_n(X) := \mathrm{Tr}(X) \cdot I_n + X \quad \forall\ X \in \mathbb{M}_n.$$

Name: Wai-Shun Cheung, University of Hong Kong.

Title: Some little results on the C-numerical ranges for rank two matrices.

Abstract: It is known that $W_A(B)$ is a (possibly degenerated) elliptical disc if $n = 2$ and that $W_A(B)$ is convex if either A or B is of rank 1. In this talk, I will present some results for matrices of rank two.

Name: Mao-Ting Chien, mtchien@scu.edu.tw

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Title: Determinantal curves and central force orbits

Abstract: Let A be an $n \times n$ matrix. The determinantal polynomial associated with A is defined by $F_A(t, x, y) = \det(t\, I_n + x\, \Re(A) + y\, \Im(A))$, where $\Re(A) = (A + A^*)/2$ and $\Im(A) = (A - A^*)/(2i)$. It is known that the numerical range of A is the convex hull of the real part of the dual curve of $F_A(t, x, y) = 0$.

A central force is a force whose magnitude depends only on the distance $|\mathbf{r}|$ of the particle from the origin: $\mathbf{f}(\mathbf{r}) = f(|\mathbf{r}|)\frac{\mathbf{r}}{|\mathbf{r}|}$. The orbit $\{(x(t), y(t)) : t_0 \le t \le t_1\}$ of a point mass under a central force $f(|\mathbf{r}|) = f(x, y)$ is described by the Newton's equations:

$$x''(t) = \frac{x(t)}{\sqrt{x(t)^2 + y(t)^2}}\, f(x, y),\ \ y''(t) = \frac{y(t)}{\sqrt{x(t)^2 + y(t)^2}}\, f(x, y).$$

In this talk, we interpret the orbit of a point mass under a central force as the algebraic curve $F_A(1, x, y) = 0$ for some matrix A.

Co-author: Hiroshi Nakazato

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Title How I could think of tensor products of matrices

Abstract In all years, I have mathematical dreams on completely positive linear maps, concerning tensor products of complex matrices (as well as the incredible structure of nuclear C^*-algebras). Suddenly, I wandered into the quantized world of fantasies and controversies. To release myself from Quantum Entanglements and the Principle of Locality, I need to seek the meaning of physics and the value of metaphysics. Conclusion: I THINK, THEREFORE I AM a pure mathematician.

Name: Hwa-Long Gau, hlgau@math.ncu.edu.tw

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Title: Crawford numbers of companion matrices

Abstract: The (generalized) Crawford number $C(A)$ of an n-by-n complex matrix A is, by definition, the distance from the origin to the boundary of the numerical range $W(A)$ of A. If A is a companion matrix

$$\begin{bmatrix} 0 & 1 & & & & \\ & 0 & 1 & & & \\ & & \ddots & \ddots & & \\ & & & \ddots & \ddots & \\ & & & & 0 & 1 \\ -a_n & -a_{n-1} & \cdots & \cdots & -a_2 & -a_1 \end{bmatrix},$$

then it is easily seen that $C(A) \geq \cos(\pi/n)$. The main purpose of this talk is to determine when the equality $C(A) = \cos(\pi/n)$ holds. A sufficient condition for this is that the boundary of $W(A)$ contains a point λ for which the subspace of $\mathbb{C}^n$ spanned by the vectors x with $\langle Ax, x\rangle = \lambda\|x\|^2$ has dimension 2, while a necessary condition is $\sum_{j=0}^{n-2} a_{n-j}e^{(n-j)i\theta} \sin\big((j+1)\pi/n\big) = \sin(\pi/n)$ for some real θ. Examples are given showing that in general these conditions are not simultaneously necessary and sufficient. We then prove that they are if A is (unitarily) reducible. We also establish a lower bound for the numerical radius $w(A)$ of A: $w(A) \geq \cos(\pi/(n+1))$, and show that the equality holds if and only if A is equal to the n-by-n Jordan block.

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Title: Construction of multi-partite genuine entanglement witnesses

Abstract: A bi-partite state ϱ in $M_A \otimes M_B$ is separable if it is a convex sum of pure product states, and a state is called entangled if it is not separable, where M_A and M_B denote matrix algebras. We may use the same definition of separability and entanglement for arbitrary multi-partite states, but we have various kinds of entanglement in these cases. For example, a tri-partite state ϱ in $M_A \otimes M_B \otimes M_C$ is said to be A-BC separable if it is separable as a bi-partite state in $M_A \otimes M_{BC}$, where M_{BC} denotes $M_B \otimes M_C$. A tri-partite state is said to be bi-separable it is the convex sum of A-BC, B-CA and C-AB separable states, and genuinely entangled if it is not bi-separable. Multi-partite genuine entanglement is defined similarly.

Distinguishing entanglement from separability is one of the main research topics in current quantum information theory. For this purpose, the duality between states and linear maps play the central role. In order to detect various kinds of multi-partite entanglement, we need also various kinds of positivity for multi- linear maps, to get appropriate entanglement witnesses. In this talk, we will exhibit two methods to construct multi-partite genuine entanglement. One of them is to use so called X-shaped Hermitian matrices, and the other is to use elementary operators and various kinds of transposes.

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Title: Gau–Wu Numbers of Nonnegative Matrices

Abstract: For any n-by-n matrix A, we consider the maximum number $k = k(A)$ of orthonormal vectors $x_j \in \mathbb{C}^n$ such that the scalar products $\langle Ax_j, x_j \rangle$ lie on the boundary $\partial W(A)$ of the numerical range $W(A)$. This number is called the Gau–Wu number of the matrix A. If A is an n-by-n $(n \geq 2)$ nonnegative matrix with the permutationally irreducible real part of the form

$$\begin{bmatrix} 0 & A_1 & & 0 \\ & 0 & \ddots & \\ & & \ddots & A_{m-1} \\ 0 & & & 0 \end{bmatrix},$$

where $m \geq 3$ and the diagonal zeros are zero square matrices, then $k(A)$ has an upper bound $m-1$. In addition, we also obtain necessary and sufficient conditions for $k(A) = m-1$ for such a matrix A. Another class of nonnegative matrices we study is the doubly stochastic ones. We prove that the value of $k(A)$ is equal to 3 for any 3-by-3 doubly stochastic matrix A. For any 4-by-4 doubly stochastic matrix, we also determine its numerical range, which is then applied to find its Gau–Wu numbers. Furthermore, the lower bound of the Gau–Wu number $k(A)$ is also found for a general n-by-n $(n \geq 5)$ doubly stochastic matrix A via the possible shapes of $W(A)$.

Name: Chi-Kwong Li, ckli@math.wm.edu

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Title: Linear algebra problems over $\mathbf{Z}_n$

Abstract: Motivated by problems in theoretical and applied topics, we consider the null spaces and column spaces of the powers of matrices over $\mathbf{Z}_n$. The results allows us to determine the number of polynomial functions on $\mathbf{Z}_n$, and check whether a function is a polynomial function. Furthermore, it allows us to determine the behavior of a dynamical systems modeled by matrices over $\mathbf{Z}_n$.

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Name: Zhongshan Li, zli@gsu.edu

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Title: Minimum ranks of sign patterns and zero-nonzero patterns and point–hyperplane configurations

Abstract: . A *sign pattern (matrix)* is a matrix whose entries are from the set $\{+,-,0\}$. The *minimum rank* (respectively, *rational minimum rank*) of a sign pattern matrix $\mathcal{A}$ is the minimum of the ranks of the real (respectively, rational) matrices whose entries have signs equal to the corresponding entries of $\mathcal{A}$. A sign pattern $\mathcal{A}$ is said to be *condensed* if $\mathcal{A}$ has no zero row or column and no two rows or columns are identical or negatives of each other. A *zero-nonzero pattern (matrix)* is a matrix whose entries are from the set $\{0,\star\}$, where $\star$ indicates a nonzero entry. Many of the sign pattern notions carry over to zero-nonzero patterns, assuming that the ground field is $\mathbb{R}$. In this paper, a direct connection between condensed $m \times n$ sign patterns and zero-nonzero patterns with minimum rank r and m point–n hyperplane configurations in $\mathbb{R}^{r-1}$ is established. In particular, condensed sign patterns (and zero-nonzero patterns) with minimum rank 3 are closely related to point–line configurations on the plane. Using this connection, we construct the smallest known sign pattern whose minimum rank is 3 but whose rational minimum rank is greater than 3. It is proved that for any sign pattern or zero-nonzero pattern $\mathcal{A}$, if the number of zero entries on each column of $\mathcal{A}$ is at most 2, then the rational and real minimum ranks of $\mathcal{A}$ are equal. Further, it is shown that for any zero-nonzero pattern $\mathcal{A}$ with minimum rank $r \geq 3$, if the number of zero entries on each column of $\mathcal{A}$ is at most $r-1$, then the rational minimum rank of $\mathcal{A}$ is also r. A few related conjectures and open problems are raised.

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Name: Yongdo Lim, ylim@skku.edu

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Title: Nonexpansive matrix means and matrix equations

Abstract: We present a constructive scheme of parametrized multivariable matrix means of positive definite matrices which interpolate the harmonic mean and the arithmetic mean, and contract the Thompson metric. Classical results on nonlinear matrix equations (e.g., Stein, Ferrante and Levy-like equations) depending on the nonexpansive property of the arithmetic average are considered in the context of the parametrized matrix means.

Name: Rajesh Pereira, pereirar@uoguelph.ca

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Title: Matrices over inclines and semirings

Abstract: A semiring is an algebraic structure which satisfies all of the properties of a ring except for the existence of inverses. An incline is a special class of semiring. We will describe characterizations of completely positive matrices over certain inclines and show that the CP-rank of these matrices satisfies the Drew-Johnson-Loewy bound. We explore practical applications of completely positive matrices over inclines. We also discuss a generalization of the determinantal rank for matrices over semirings.

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Title: Spectra and variance of quantum random variables

Abstract: The matricial range and the matricial spectrum are matrix-valued analogues of the numerical range and spectrum of a matrix or operator. In my lecture I will explain how the application of these concepts to quantum random variables leads to new ways of understanding expected value and variance in the noncommutative setting.

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Title: Pseudospectra of special operators and Pseudosectrum preservers

Abstract: Denote by $\mathcal{B}(H)$ the Banach algebra of all bounded linear operators on a complex Hilbert space H. Let $A \in \mathcal{B}(H)$, and denote by $\sigma(A)$ the spectrum of A. For $\varepsilon > 0$, define the ε-pseudospectrum $\sigma_\varepsilon(A)$ of A as

$$\sigma_\varepsilon(A) = \{z \in \sigma(A+E) : E \in \mathcal{B}(H),\ \|E\| < \varepsilon\}.$$

In this paper, the pseudospectra of several special classes of operators are characterized. As applications, complete descriptions are given to the maps on operators leaving invariant the pseudospectra of $A \bullet B$ for different kind of binary operation $\bullet$ on operators such as the difference $A - B$, the operator product AB, and the Jordan product $AB + BA$.

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Name: Raymond Nung-Sing Sze, raymond.sze@polyu.edu.hk

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Title: Homomorphisms on the semigroup of positive matrices

Abstract: In this talk, we will give a characterization of the homomorphism on the semigroup of (entrywise) positive matrices. It is showed that either the homomorphism has of the form $A \mapsto SAS^{-1}$ for some nonnegative monomial matrix S or its restricted map on the set of singular positive matrices is a constant map.

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Title: The Jordan form of an irreducible eventually nonnegative matrix

Abstract: A square complex matrix A is said to be eventually nonnegative if there exists a positive integer k_0 such that for all $k \geq k_0$, $A^k \geq 0$; A is strongly eventually nonnegative if it is eventually nonnegative and has an irreducible nonnegative power.

Let $\mathcal{J}$ be a multiset of elementary Jordan blocks. The radius of $\mathcal{J}$ is $\rho(\mathcal{J}) := \max\{|\lambda| : J_k(\lambda) \in \mathcal{J}\}$ and the boundary of $\mathcal{J}$ is $\partial(\mathcal{J}) := \{J_k(\lambda) \in \mathcal{J} : |\lambda| = \rho(\mathcal{J})\}$. We say $\mathcal{J}$ is a Frobenius Jordan

multiset if (1) $\rho(\mathcal{J}) > 0$, (2) $\partial(\mathcal{J}) = \{J_1(\rho(\mathcal{J})\omega^j) : j = 0, \ldots, r-1\}$, where r is the cardinality of $\partial(\mathcal{J})$ and $\omega = e^{2\pi i/r}$, and (3) $\omega\mathcal{J} = \mathcal{J}$, where $\omega\mathcal{J}$ is the multiset of Jordan blocks $J_k(\omega\lambda)$, where $J_k(\lambda)$ ranges over the elements of $\mathcal{J}$. $\mathcal{J}$ is self-conjugate if $\bar{\mathcal{J}} = \mathcal{J}$, where $\bar{\mathcal{J}}$ is the multiset of elementary Jordan blocks $J_k(\bar{\lambda})$, where $J_k(\lambda)$ ranges over the elements of $\mathcal{J}$. If $\rho(\mathcal{J}) > 0$, then we say $\mathcal{J}$ is of cyclic index r if r is the largest integer such that $e^{2\pi i/r}\mathcal{J} = \mathcal{J}$. By the cyclic index of a square complex matrix A we mean the largest integer m such that the digraph of A is cyclically m-partite.

Theorem 1. If A is an irreducible eventually nonnegative matrix with cyclic index r and $\text{rank}A^2 = \text{rank}A$, then $\mathcal{J}(A)$, the multiset of elementary Jordan blocks of A, is a self-conjugate Frobenius Jordan multiset with cyclic index r.

Theorem 1 answers in the affirmative an open question raised by Zaslavsky and Tam.

Theorem 2. If A is irreducible and eventually nonnegative and $\text{rank}A^2 = \text{rank}A$, then A is strongly eventually nonnegative.

We give an example of an irreducible eventually nonnegative matrix A with $\rho(A) > 0$ (and $\text{rank}A^2 \neq \text{rank}A$) such that the elementary Jordan blocks of A associated with $\rho(A)$ are not all 1×1; hence A is not strongly evenutally nonnegative. This answers in the negative an open question raised by Tam and Zaslavsky.

Theorem 3. A multiset of elementary Jordan blocks $\mathcal{J}$ is a self-conjugate Frobenius multiset with cyclic index r if and only if there exists a strongly eventually nonnegative matrix A with r dominant eigenvalues such that $\mathcal{J}(A) = \mathcal{J}$.

Co-author(s): Leslie Hogben, Ulrica Wilson

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Title: Multiplicities and Numerical Ranges

Abstract: We will discuss the multiplicity of a point in the numerical range. Similar concepts and results will be presented for the generalized numerical ranges.

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Title: Maps preserving determinants of convex combinations

Abstract: Suppose a map ϕ satisfies $\det(A + B) = \det(\phi(A) + \phi(B))$ for any positive definite matrices A and B, then we have $tr(AB^{-1}) = tr(\phi(A)\phi(B)^{-1})$. Through this viewpoint, we show that ϕ is of the form $\phi(A) = MAM^*$ or $\phi(A) = MA^tM^*$ for some invertible matrix M with $\det(M^*M) = 1$. We also characterize the map $\phi : S \to S$ preserving the determinants of convex combinations in S using the similar method. Here S can be complex matrices, positive definite matrices, symmetric matrices, and upper triangular matrices.

Co-authors: Huajun Huang, Chih-Neng Liu, Patrícia Szokol, and Jun Zhang

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Title: Random Matrix and Preconditioning.

Abstract: In this talk, we establish the algebraic properties of random Toeplitz operators following Brown and Halmos, and the Fourier theory of Toeplitz forms following Szegö. We study the distribution of the eigenvalue functions of a random Toeplitz matrix and the preconditioning of a random Toeplitz operator by the Strang circulants following Raymond Chan. This is to be applied in solving random Toeplitz systems $T\mathbf{x} = \mathbf{b}$ by the preconditioned conjugate gradient method. Numerical examples are given.

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Title: On graphs whose maximum nullity equals zero forcing number

Abstract: Given a graph G on n vertices and a field F, the maximum nullity of G over F, denoted by $M^F(G)$, is the largest possible nullity over all $n \times n$ symmetric matrices $A = [a_{ij}]$ over F where for any $i \neq j$, $a_{ij} \neq 0$ if and only if ij is an edge in G. The zero forcing number $Z(G)$ of G was introduced in [AIM Minimum Rank–Special Graphs Work Group, Linear Algebra and its Applications, 428 (2008) 1628-1648] to bound $M^F(G)$ from above. In this talk, we give an affirmative answer to Conjecture 2.5 in [M. Catral et al., Electronic Journal of Linear Algebra, 23 (2012) 906-933] by proving a general upper bound on $Z(G)$ for any graph G. Our results generalize the main results appeared in [W. Barrett et al. Electronic Journal of Linear Algebra, 27 (2014) 444-457]. In this talk, we also discuss the relationship between $M^F(G)$ and $Z(G)$ for two other graph classes related to subdivision graphs.

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Title: Combinatorics of Stochastic Cubes

Abstract: The celebrated Birkhoff- von Neumann theorem on the polytope of doubly stochastic matrices states that the polytope is generated by the 0-1 permutation matrices. That is, the 0-1 permutation matrices are the extreme points (vertices) of the convex compact set of doubly stochastic matrices. We study combinatorial properties and structure of stochastic cubes (a.k.a. stochastic tensors or semi-magic cubes) in quest of the counterparts of the Birkhoff – von Neumann result in higher dimensions.