On finite groups in which elements of the same order are conjugate

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Let $a, b \in G$ be conjugate elements in a finite group G, i.e. there exists $g \in G$ such that $g^{-1}ag = b$, it's obvious that a and b have the same order. But $a, b \in G$ with the same order maybe not conjugate in G. For example: 1,2 are elements of the same order 3 in $\mathbb{Z}_3$, but there exists no $g \in \mathbb{Z}_3$ such that $g^{-1}ag = b$ since $\mathbb{Z}_3$ is Abelian.

Problem Determine all finite groups in which elements of the same order are conjugate.

Solution $G \cong S_1, S_2$ or S_3.

Reference

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