Computation of determinant

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The theory of determinants is well-known and provides important information in many fields see [1]. In fact, it continues to some generate elegant and challenging problems, [2].

There are various ways to define the determinant of a square matrix in [1]. Recall that the determinant of an n-by-n matrix $A = (a_{i,j})$ is defined as

$$det(A) \equiv \sum_{\pi \in S_n} (-1)^{\mathcal{I}(\pi)} \prod_{i=1}^{n} a_{i,\pi(i)} \tag{1}$$

where S_n (also known as the symmetric group on n elements) is the set of all permutations of $\{1, 2, \ldots, n\}$, i.e., all the ways of pairing up the n rows of the matrix with the n columns, and $\mathcal{I}(\pi)$ is the inversion number of π, the minimal number of transpositions of adjacent columns needed to turn π into the identity permutation. This formula [1] is practical for some simple matrices such as 3-by-3 and 4-by-4 matrices, but for large dense matrices it is inefficient, since there have many of computation.

At present, most mathematicians are familiar with **Gaussian elimination** as a more practical method of evaluating determinants, while there have many of filled elements for a sparse matrix.

Problem: The problem is how to deal with the algebraic expression (1) to reduce its computation and at the same time increase its accuracy for some special matrices.

References

[1] Wikipedia, the free encyclopedia—http://en.wikipedia.org/wiki/Determinant.

[2] Greg Kuperberg, Another proof of the alternating sign matrix conjecture, Internat. Math. Res. Notices, 1996(3): 139-150. (arXiv:math/9712207)

[3] Anne-Ly Do, S. Boccaletti and T. Gross, Graphical notation reveals topological stability criteria for collective dynamics in complex networks, Phy. Rev. Lett., 2012(108), 194102.