Sufficient Condition for AB = BA
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Let $\mathcal{H}$, $\mathcal{B}(\mathcal{H})$ be separable complex Hilbert space and the set of all bounded linear operators on $\mathcal{H}$, respectively. The elements of $\mathcal{E}(\mathcal{H}) = \{A \in \mathcal{B}(\mathcal{H}) : 0 \leq A \leq I\}$ are called quantum effects. For $A, B \in \mathcal{E}(\mathcal{H})$, the sequential product of A and B is $A \circ B = A^{\frac{1}{2}} B A^{\frac{1}{2}}$ and the Jordan product of A and B is $A * B = \frac{AB+BA}{2}$. Many of results show that algebraic conditions on $A \circ B$ or $A * B$ imply that $AB = BA$ [?]-[?]. There are some questions as following.

1. Can we get that $A_i, 1 \leq i \leq n$ are commutative if sequential product $A_n \circ A_{n-1} \circ \cdots \circ A_2 \cdots A_1 = A_n^{\frac{1}{2}} A_{n-1}^{\frac{1}{2}} \cdots A_2^{\frac{1}{2}} A_1 A_2^{\frac{1}{2}} \cdots A_{n-1}^{\frac{1}{2}} A_n^{\frac{1}{2}}$ satisfies certain distributive or associative laws.

2. Can we get that $A_i, 1 \leq i \leq n$ are commutative if sequential product $A_n * A_{n-1} * \cdots * A_2 * A_1$ satisfies certain distributive or associative laws.

References

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