

## Decomposition of Tiffoli Gate

**Problem** Show that one cannot decompose  $A = I_7 \oplus [-1]$  as a product of 4 unitary matrices  $U_1 U_2 U_3 U_4$ , where for each  $j = 1, 2, 3, 4$ ,  $U_j$  has one of the following form:

$$I_2 \otimes V, \quad P(I_2 \otimes V)P^t, \quad V \otimes I_2,$$

where  $V \in M_4$  is unitary, and  $P = I_2 \oplus \begin{bmatrix} 0 & I_2 \\ I_2 & 0 \end{bmatrix} \oplus I_2$ .

## Reference

N. Yu, R. Duan, and M. Ying, Five two-qubit gates are necessary for implementing Toffoli gate, arXiv:1301.3372v1.