IBM Standard Gates

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1. Pauli Matrix $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
2. Pauli Matrix $Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
3. Pauli Matrix $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
4. Hadamard Gate $H = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$
5. $S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$
6. $T = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i \end{pmatrix}$
7. $U_1(\lambda) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\lambda} \end{pmatrix}$
8. $U_2(\lambda, \phi) = \begin{pmatrix} 1 & e^{-i\lambda} \\ e^{i\phi} & e^{i(\phi+\lambda)} \end{pmatrix}$
9. $U_3(\lambda, \phi, \theta) = \begin{pmatrix} \cos(\frac{\theta}{2}) & e^{-i\lambda}\sin(\frac{\theta}{2}) \\ e^{i\phi}\cos(\frac{\theta}{2}) & e^{i(\phi+\lambda)}\cos(\frac{\theta}{2}) \end{pmatrix}$
10. $Rx(\alpha) = \begin{pmatrix} \cos(\frac{\beta}{2}) & -i\sin(\frac{\beta}{2}) \\ -i\sin(\frac{\beta}{2}) & \cos(\frac{\beta}{2}) \end{pmatrix} = e^{-i\frac{\gamma}{2}X}$
11. $Ry(\beta) = \begin{pmatrix} \cos(\frac{\beta}{2}) & -\sin(\frac{\beta}{2}) \\ \sin(\frac{\beta}{2}) & \cos(\frac{\beta}{2}) \end{pmatrix} = e^{-i\frac{\gamma}{2}Y}$
12. $Rz(\gamma) = \begin{pmatrix} e^{-i\frac{\gamma}{2}} & 0 \\ 0 & e^{i\frac{\gamma}{2}} \end{pmatrix} = e^{-i\frac{\gamma}{2}Z}$
    (?) *Qskit seems to implement this like* $U_1(\gamma)$
13. SWAP GATE $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
14. $CNOT = I_2 \oplus X$
15. $CZ = I_2 \oplus Z$
16. $CH = I_2 \oplus H$
17. $CRz = I_2 \oplus R_z$
18. Toffoli Gate or $CCNOT = I_6 \oplus X$

Shorter Basic Gate Decomposition

We can also decompose U as follows:

$$U = \left(\begin{array}{cccc|cccc} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \sqrt{2/3} & 0 & 0 & 0 & 0 & \sqrt{1/3} & 0 & 0 \\ -\sqrt{1/6} & 0 & \sqrt{1/2} & 0 & 0 & \sqrt{1/3} & 0 & 0 \\ 0 & \sqrt{1/6} & 0 & \sqrt{1/2} & 0 & 0 & \sqrt{1/3} & 0 \\ \hline -\sqrt{1/6} & 0 & -\sqrt{1/2} & 0 & 0 & \sqrt{1/3} & 0 & 0 \\ 0 & \sqrt{1/6} & 0 & -\sqrt{1/2} & 0 & 0 & \sqrt{1/3} & 0 \\ 0 & -\sqrt{2/3} & 0 & 0 & 0 & 0 & \sqrt{1/3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right) = C_1 \tilde{C}_2 \tilde{C}_3 P_2 \tilde{C}_4 \tilde{C}_5 \tilde{P}_1 \tilde{C}_3$$

where $C_1 = \begin{pmatrix} 0 & 0 & I_2 & 0 \\ 0 & I_2 & 0 & 0 \\ I_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_2 \end{pmatrix}$, $\quad \tilde{C}_2 = \left[\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \otimes I_2\right] \oplus I_4$, $\quad \tilde{C}_3 = (I_2 \otimes \sigma_x) \oplus I_4$,

$P_2 = I_4 \oplus \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, $\quad \tilde{C}_4 = \begin{pmatrix} \sigma_z & 0 & 0 & 0 \\ 0 & I_2 & 0 & 0 \\ 0 & 0 & \sigma_z & 0 \\ 0 & 0 & 0 & I_2 \end{pmatrix}$, $\quad \tilde{C}_5 = \begin{pmatrix} \sqrt{\frac{1}{3}} I_2 & 0 & -\sqrt{\frac{2}{3}} I_2 & 0 \\ 0 & I_2 & 0 & 0 \\ -\sqrt{\frac{2}{3}} I_2 & 0 & -\sqrt{\frac{1}{3}} I_2 & 0 \\ 0 & 0 & 0 & I_2 \end{pmatrix}$,

and $\tilde{P}_1 = I_4 \oplus \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

Matlab Verification

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sx=[0,1;1,0];
sz=[1,0;0,-1];
e0=[1,0;0,0];
e1=[0,0;0,1];
W1=[sqrt(1/2),-sqrt(1/2);sqrt(1/2),sqrt(1/2)];
W2=[sqrt(1/3),-sqrt(2/3);-sqrt(2/3),-sqrt(1/3)];
U=[0,0,0,0,1,0,0,0; sqrt(2/3),0,0,0,0,sqrt(1/3),0,0;
   -sqrt(1/6),0,sqrt(1/2),0,0,sqrt(1/3),0,0; 0,sqrt(1/6),0,sqrt(1/2),0,0,sqrt(1/3),0;
   -sqrt(1/6),0,-sqrt(1/2),0,0,sqrt(1/3),0,0; 0,sqrt(1/6),0,-sqrt(1/2),0,0,sqrt(1/3),0;
   0,-sqrt(2/3),0,0,0,0,sqrt(1/3),0;0,0,0,0,0,0,0,1];

C1=kron(kron(sx,e0),eye(2))+kron(kron(eye(2),e1),eye(2))
C2=kron(kron(e0,W1),eye(2))+kron(kron(e1,eye(2)),eye(2))
C3=kron(kron(e0,eye(2)),sx)+kron(kron(e1,eye(2)),eye(2))
P2=[eye(4),zeros(4,4);zeros(4,4),[0,0,1,0;0,1,0,0;1,0,0,0;0,0,0,1]]
C4=kron(kron(eye(2),e0),sz)+kron(kron(eye(2),e1),eye(2))
C5=kron(kron(W2,e0),eye(2))+kron(kron(eye(2),e1),eye(2))
P1=[eye(4),zeros(4,4);zeros(4,4),[0,0,-1,0;0,1,0,0;1,0,0,0;0,0,0,1]]

C1*C2*C3*P2*C4*C5*P1*C3-U %must be approximately the zero 8x8 matrix
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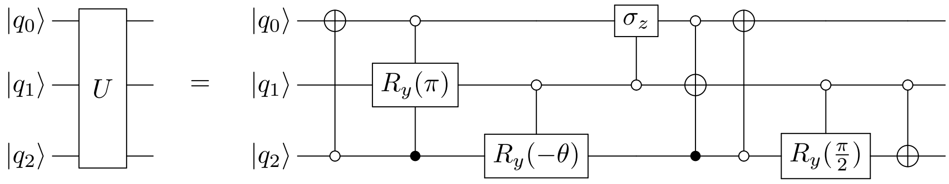
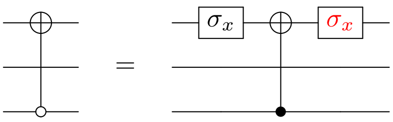
Circuit Diagrams

The decomposition above is illustrated here in a circuit diagram

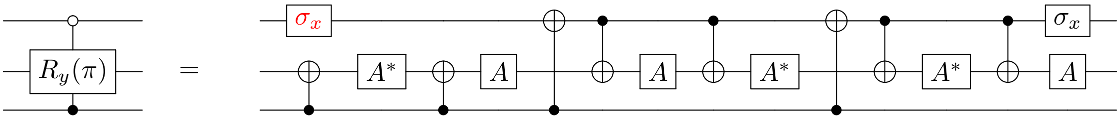


Let $A = R_y(\frac{\pi}{4})$ and $\theta = 2\arccos(-\sqrt{\frac{2}{3}})$. Note that we can further decompose the above gates into basic gates (CNOTS and single qubit gates) as follows

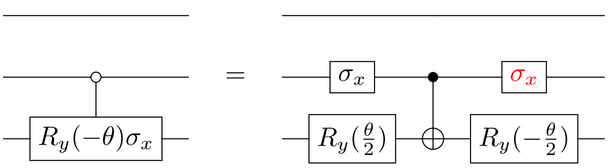
$\tilde{C}_3$:



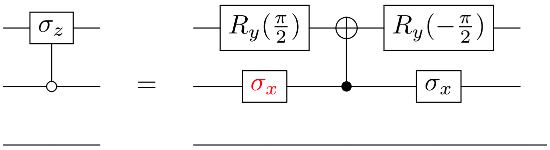
$\tilde{P}_1$:



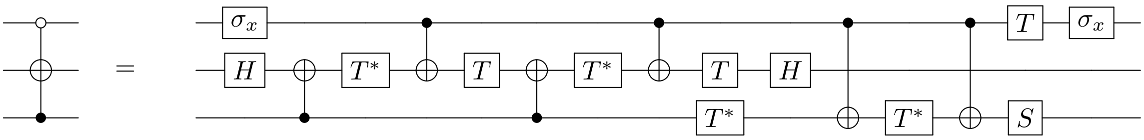
$\tilde{C}_5$:



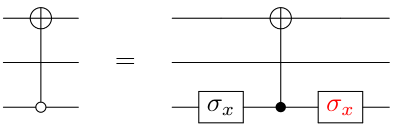
$\tilde{C}_4$:



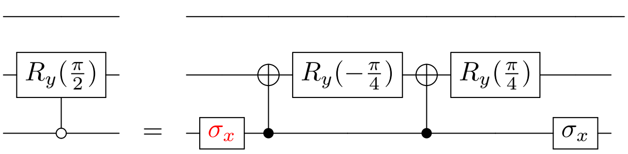
P_2 :



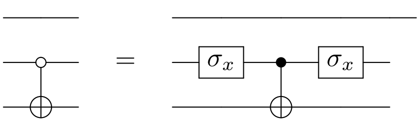
$\tilde{C}_3$:



$\tilde{C}_2$:



C_1 :



Thus, U can be decomposed into a circuit consisting of 21 CNOT gates (of which 6 have nonadjacent control and target bits) and 26-32 single qubit gates (we can combine some of the single qubit gates).