

1 (5 points) Find the point of intersection of the lines: $x_1 + 2x_2 = -13$, $3x_1 - 2x_2 = 1$.

Solution. To find the point of intersection, we solve the system with augmented matrix

$$\begin{bmatrix} 1 & 2 & -13 \\ 3 & -2 & 1 \end{bmatrix}.$$

By row reduction, we get

$$\begin{bmatrix} 1 & 2 & -13 \\ 3 & -2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \end{bmatrix}.$$

Then ...

2 (5 points) Solve the linear system:

$$\begin{aligned} x_1 - 5x_2 + 4x_3 &= -3 \\ 2x_1 - 7x_2 + 3x_3 &= -2 \\ -2x_1 + x_2 + 7x_3 &= -1 \end{aligned}$$

Solution. Consider augmented matrix

$$\begin{bmatrix} 1 & -5 & 4 & -3 \\ 2 & -7 & 3 & -2 \\ -2 & 1 & 7 & -1 \end{bmatrix}.$$

By row reduction, we get

3 (5 points) Solve the linear system with augmented matrix

$$\begin{bmatrix} 2 & 0 & -6 & -8 \\ 0 & 1 & 2 & 3 \\ 3 & 6 & -2 & -4 \end{bmatrix}.$$

Solution. By row reduction, we get

4 (5 points) Find all the values h so that the linear system with the following augmented matrix is consistent: Applying row reduction to:

$$\begin{bmatrix} 1 & h & -5 \\ 2 & -8 & 6 \end{bmatrix}.$$

Solution. By row reduction, we get ...

So, the system is consistent exactly when h satisfies ...

5 (5 points) Row reduce the following augmented matrix. Determine the leading ones and pivot columns.

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 4 & 5 & 4 & 3 \end{bmatrix}.$$

Solution.

6 (5 points) Find the solutions of the system with augmented matrix:

$$\begin{bmatrix} 1 & -2 & -1 & 4 \\ -2 & 4 & -5 & 6 \end{bmatrix}.$$

7 (5 points) Find the solutions of the system with augmented matrix:

$$\begin{bmatrix} 1 & 0 & -9 & 0 & 4 \\ 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

8 (5 points) Determine the conditions on (h, k) so that the following system has no solution, one solution, and infinitely many solutions.

$$x_1 - 3x_2 = 1, \quad 2x_1 + hx_2 = k.$$

Solution. Row reduce the augmented matrix...