Math 211 Homework 1

**Name: Sample Solution**

**1** (5 points) Find the point of intersection of the lines: $x_1 + 2x_2 = -13, 3x_1 - 2x_2 = 1$.
Solution. Apply row operation to the augmented matrix

$$\begin{bmatrix} 1 & 2 & -13 \\ 3 & -2 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -13 \\ 0 & -8 & 40 \end{bmatrix}.$$

Back substitution yields $x_2 = -5, x_1 = -13 - 2(-5) = -3$. Thus, $(x_1, x_2) = (-3, -5)$.

**2** (5 points) Solve the linear system:

$$\begin{aligned} x_1 - 5x_2 + 4x_3 &= -3 \\ 2x_1 - 7x_2 + 3x_3 &= -2 \\ -2x_1 + x_2 + 7x_3 &= -1 \end{aligned}$$

Solution. Apply row operations to the augmented matrix

$$\begin{bmatrix} 1 & -5 & 4 & -3 \\ 2 & -7 & 3 & -2 \\ -2 & 1 & 7 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & -5 & 4 & -3 \\ 2 & -7 & 3 & -2 \\ 0 & -6 & 10 & -3 \end{bmatrix} \to \begin{bmatrix} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & -6 & 10 & -3 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & -5 & 4 & -3 \\ 2 & -7 & 3 & -2 \\ 0 & -6 & 10 & -3 \end{bmatrix} \to \begin{bmatrix} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & 0 & 0 & 5 \end{bmatrix}.$$

So, there the system is not consistent, i.e., no solution.

**3** (5 points) Solve the linear system with augmented matrix

$$\begin{bmatrix} 2 & 0 & -6 & -8 \\ 0 & 1 & 2 & 3 \\ 3 & 6 & -2 & -4 \end{bmatrix}.$$

Solution. Apply row operations to the augmented matrix

$$\begin{bmatrix} 2 & 0 & -6 & -8 \\ 0 & 1 & 2 & 3 \\ 3 & 6 & -2 & -4 \end{bmatrix} \to \begin{bmatrix} 2 & 0 & -6 & -8 \\ 0 & 1 & 2 & 3 \\ 0 & 6 & 7 & 8 \end{bmatrix} \to \begin{bmatrix} 2 & 0 & -6 & -8 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 5 & 10 \end{bmatrix}.$$

Back substitution yields, $x_3 = 2, x_2 = -1, x_1 = 2$.

**4** (5 points) Find all the values $h$ so that the linear system with the following augmented matrix is consistent:Applying row reduction to:

$$\begin{bmatrix} 1 & h & -5 \\ 2 & -8 & 6 \end{bmatrix}.$$

Solution. Apply row reduction to the augmented matrix

$$\begin{bmatrix} 1 & h & -5 \\ 2 & -8 & 6 \end{bmatrix} \to \begin{bmatrix} 1 & -4 & 3 \\ 1 & h & -5 \end{bmatrix} \to \begin{bmatrix} 1 & -4 & 3 \\ 0 & h+4 & -8 \end{bmatrix}.$$

So, the system is consistent when $h \neq -4$, and the solution is $x_2 = -8/(h+4), x_1 = 3 + 4x_2 = -32/(h+4)$.

**5** (5 points) Row reduce the following augmented matrix. Determine the leading ones and pivot columns.

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 4 & 5 & 4 & 3 \end{bmatrix}.$$

Solution. By row reduction,

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 4 & 5 & 4 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 0 & -3 & -6 & -5 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 0 & -3 & -6 \\ 0 & 1 & 2 & 5/3 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 2 & 5/3 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 1 & 2 & 5/3 \\ 0 & 0 & 1 & 2 \end{bmatrix}.$$

So, Columns 1,2,3 are pivot columns, and $(1,1),(2,2),(3,3)$ entries are pivot positions.

**6** (5 points) Find the solutions of the system with augmented matrix:

$$\begin{bmatrix} 1 & -2 & -1 & 4 \\ -2 & 4 & -5 & 6 \end{bmatrix}.$$

Solution. Apply row reduction to the augmented matrix

$$\begin{bmatrix} 1 & -2 & -1 & 4 \\ -2 & 4 & -5 & 6 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & -1 & 4 \\ 0 & 0 & 7 & -14 \end{bmatrix}.$$

Back substitution yields, $x_3 = -2$, $x_2 = t$, $x_1 = 4 + x_3 + 2x_2 = 2 + 2t$ with $t \in \mathbf{R}$.

**7** (5 points) Find the solutions of the system with augmented matrix:

$$\begin{bmatrix} 1 & 0 & -9 & 0 & 4 \\ 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Solution. Back substitution yields $x_4 = -7, x_3 = t, x_2 = -1 - 3t, x_1 = 4 + 9t$ with $t \in \mathbf{R}$.

**8** (5 points) Determine the conditions on $(h,k)$ so that the following system has no solution, one solution, and infinitely many solutions.

$$x_1 - 3x_2 = 1, \quad 2x_1 + hx_2 = k.$$

Solution. Apply row reduction to the augmented matrix

$$\begin{bmatrix} 1 & -3 & 1 \\ 2 & h & k \end{bmatrix} \to \begin{bmatrix} 1 & -3 & 1 \\ 0 & h+6 & k-2 \end{bmatrix}.$$

So, the system is inconsistent when $h = -6$ and $k \neq 2$; it has infinitely many solution if $h = -6$ and $k = 2$; it has a unique solution if $h \neq -6$.