1 (5 points) Write the augmented matrix of a system of linear equations for the following vector equation
\[
\begin{bmatrix}
3 & -2 \\
7 & 3 \\
-2 & 1
\end{bmatrix} + \begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]
and solve the system.

2 (5 points) Write the following linear system as vector equation,
\[
\begin{align*}
3x_1 - 2x_2 + 4x_3 &= 3, \\
-2x_1 - 7x_2 + 5x_3 &= 1, \\
5x_1 + 4x_2 - 3x_3 &= 2,
\end{align*}
\]
and solve the system.

3 (5 points) Find all the values of \(h\) such that the following vector equation has solution(s),
\[
\begin{bmatrix}
1 \\
0 \\
-2
\end{bmatrix} + \begin{bmatrix}
x_2 \\
1 \\
7
\end{bmatrix} = \begin{bmatrix}
h \\
-3 \\
-5
\end{bmatrix}.
\]

4 (10 points) Given the following matrix \(A\) and a vector \(b\).
\[
A = \begin{bmatrix}
2 & 0 & 6 \\
-1 & 8 & 5 \\
1 & -2 & 1
\end{bmatrix}, \quad b = \begin{bmatrix}
10 \\
3 \\
7
\end{bmatrix}.
\]
(a) Determine if \(b\) is a linear combination of the columns of \(A\).
(b) Show that the second column of \(A\) is a linear combination of the columns of \(A\).
[No calculation is needed in (b).]

5 (5 points) Compute the matrix product \(Ab\) for
\[
A = \begin{bmatrix}
1 & 3 & -4 \\
3 & 2 & 1
\end{bmatrix}, \quad b = \begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix}.
\]

6 (5 points) Let \(A = \begin{bmatrix}
1 & -2 & -1 \\
-2 & 2 & 0 \\
4 & -1 & 3
\end{bmatrix}\). Show that \(Ax = b\) is not always solvable for all \(b \in \mathbb{R}^3\), and describe the set of \(b \in \mathbb{R}^3\) (as a plane or a line) such that the system is solvable.

7 (5 points) Given three vectors
\[
\begin{align*}
v_1 &= \begin{bmatrix}
0 \\
0 \\
-3
\end{bmatrix},
v_2 &= \begin{bmatrix}
0 \\
-3 \\
9
\end{bmatrix},
v_3 &= \begin{bmatrix}
4 \\
-2 \\
-6
\end{bmatrix},
\end{align*}
\]
determine if these vectors generate, or span, \(\mathbb{R}^3\).

8 (5 points) Given that
\[
\begin{align*}
u &= \begin{bmatrix}
7 \\
2 \\
5
\end{bmatrix}, v &= \begin{bmatrix}
3 \\
1 \\
3
\end{bmatrix}, w &= \begin{bmatrix}
5 \\
1 \\
1
\end{bmatrix} \quad \text{and} \quad 2u - 3v - w = 0.
\end{align*}
\]
Find $x_1$ and $x_2$ that satisfy
\[
\begin{bmatrix}
7 & 3 \\
2 & 1 \\
5 & 3
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} =
\begin{bmatrix}
5 \\
1 \\
1
\end{bmatrix}.
\]

9 (5 points) Suppose the solution set of a linear system are described as:

\[x_1 = 5x_4, x_2 = 3 - 2x_4, x_3 = 2 + 5x_4, \quad x_4 \in \mathbb{R}.
\]

Describe the solution set as a line in $\mathbb{R}^4$ in the format of $\{p + tv : t \in \mathbb{R}\}$.

10 (5 points) Describe geometrically (a line, a plane, etc.) the solution set of $x_1 - 2x_2 + 3x_3 = 0$, and the solution set of $x_1 - 2x_2 + 3x_3 = 4$, as subsets in $\mathbb{R}^3$.

11 (5 points) **Extra credits** Show that the reduced row echelon form of an augmented matrix is unique.

[Hint: Suppose $A_1, A_2$ are two different reduced row echelon forms of an augmented matrix $A$. Consider the first row from the bottom that the two matrices $A_1, A_2$ are different. Argue that they will yield different solutions...]