Math 211 Homework 2 **Name: TeX file**

1 (5 points) Write the augmented matrix of a system of linear equations for the following vector equation

$$x_1 \begin{bmatrix} 3 \\ -2 \end{bmatrix} + x_2 \begin{bmatrix} 7 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and solve the system.

2 (5 points) Write the following linear system as vector equation,

$$3x_1 - 2x_2 + 4x_3 = 3,$$

$$-2x_1 - 7x_2 + 5x_3 = 1,$$

$$5x_1 + 4x_2 - 3x_3 = 2,$$

and solve the system.

3 (5 points) Find all the values of h such that the following vector equation has solution(s),

$$x_1 \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix} = \begin{bmatrix} h \\ -3 \\ -5 \end{bmatrix}.$$

4 (10 points) Given the following matrix A and a vector $\mathbf{b}$.

$$A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 10 \\ 3 \\ 7 \end{bmatrix}.$$

(a) Determine if $\mathbf{b}$ is a linear combination of the columns of A.

(b) Show that the second column of A is a linear combination of the columns of A.
[No calculation is needed in (b).]

5 (5 points) Compute the matrix product $A\mathbf{b}$ for

$$A = \begin{bmatrix} 1 & 3 & -4 \\ 3 & 2 & 1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

6 (5 points) Let $A = \begin{bmatrix} 1 & -2 & -1 \\ -2 & 2 & 0 \\ 4 & -1 & 3 \end{bmatrix}$. Show that $A\mathbf{x} = \mathbf{b}$ is not always solvable for all $\mathbf{b} \in \mathbb{R}^3$, and describe the set of $\mathbf{b} \in \mathbb{R}^3$ (as a plane or a line) such that the system is solvable.

7 (5 points) Given three vectors

$$\mathbf{v_1} = \begin{bmatrix} 0 \\ 0 \\ -3 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 0 \\ -3 \\ 9 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} 4 \\ -2 \\ -6 \end{bmatrix},$$

determine if these vectors generate, or span, $\mathbb{R}^3$.

8 (5 points) Given that

$$\mathbf{u} = \begin{bmatrix} 7 \\ 2 \\ 5 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}, \ \mathbf{w} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad 2\mathbf{u} - 3\mathbf{v} - \mathbf{w} = 0.$$

Find x_1 and x_2 that satisfy

$$\begin{bmatrix} 7 & 3 \\ 2 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}.$$

9 (5 points) Suppose the solution set of a linear system are described as:

$$x_1 = 5x_4, x_2 = 3 - 2x_4, x_3 = 2 + 5x_4, \qquad x_4 \in \mathbb{R}.$$

Describe the solution set as a line in $\mathbb{R}^4$ in the format of $\{\mathbf{p} + t\mathbf{v} : t \in \mathbb{R}\}$.

10 (5 points) Describe geometrically (a line, a plane, etc.) the solution set of $x_1 - 2x_2 + 3x_3 = 0$, and the solution set of $x_1 - 2x_2 + 3x_3 = 4$, as subsets in $\mathbb{R}^3$.

---

11 (5 points) **Extra credits** Show that the reduced row echelon form of an augmented matrix is unique.

[Hint: Suppose A_1, A_2 are two different reduced row echelon forms of an augmented matrix A. Consider the first row from the bottom that the two matrices A_1, A_2 are different. Argue that they will yield different solutions....]