Five points for each question.

1. Determine whether the following three vectors are linearly independent.

$$\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -8 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}.$$

2. Determine the value(s) h such that the following three vectors are linearly independent.

$$\mathbf{v_1} = \begin{bmatrix} 3 \\ -6 \\ 1 \end{bmatrix}, \ \mathbf{v_2} = \begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix}, \ \mathbf{v_3} = \begin{bmatrix} 9 \\ h \\ 3 \end{bmatrix}.$$

3. Determine (with explanation) the reduced echelon form of a 3×3 matrix A if it has linearly independent columns.

4. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}, \mathbf{u} = \begin{bmatrix} 3 \\ 6 \\ -9 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

Determine $T(\mathbf{u})$ and $T(\mathbf{v})$ if $T : \mathbb{R}^3 \to \mathbb{R}^3$ is the linear transformation defined by $T(\mathbf{x}) = A\mathbf{x}$.

5. Let

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & -3 \\ 2 & -5 & 6 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -6 \\ -4 \\ -5 \end{bmatrix}.$$

Find a vector $\mathbf{x}$ such that $A\mathbf{x} = \mathbf{b}$.

6. Let

$$A = \begin{bmatrix} 3 & 2 & 10 & -6 \\ 1 & 0 & 2 & -4 \\ 0 & 1 & 2 & 3 \\ 1 & 4 & 10 & 8 \end{bmatrix}.$$

Find all vectors that are mapped into the zero vector by the transformation $\mathbf{x} \to A\mathbf{x}$.

7. Let

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \mathbf{v_1} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 7 \\ -2 \end{bmatrix},$$

and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that $T(\mathbf{x}) = x_1\mathbf{v_1} + x_2\mathbf{v_2}$. Find the matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ for each $\mathbf{x} \in \mathbb{R}^2$.

8. Find the standard matrix of a transformation T that reflects points through the horizontal x_1 axis and then reflects the point through the line $x_2 = x_1$.

[Hint: Determine $T(\mathbf{e}_1), T(\mathbf{e}_2)$).

9. Suppose $T(x_1, x_2) = (x_1 + 4x_2, 0, x_1 - 3x_2, x_1)$. Find a matrix A such that

$$A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + 4x_2 \\ 0 \\ x_1 - 3x_2 \\ x_1 \end{bmatrix}.$$

[Hint: Study Example 5 in Section 1.9.]