Math 211 Homework 4 Name:

Five points for each question unless specified otherwise.

1. Compute AB if

$$A = \begin{bmatrix} 4 & -3 \\ -3 & 5 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix}.$$

2. Construct nonzero 3×3 matrices A, B, C such that $AB = AC$ and $B \neq C$.
[Hint: You can construct A, B, C that only has one nonzero entry in each of the matrices].

3. Consider system of equations

$$7x_1 + 3x_2 = -9$$

$$-6x_1 - 3x_2 = 4$$

(a) Create a matrix equation of the form $A\mathbf{x} = \mathbf{b}$ by specifying A and $\mathbf{b}$.
(b) Compute A^{-1} using the algorithm in Section 2.3.
(c) Compute $\mathbf{x}_0 = A^{-1}\mathbf{b}$ and verify that $A\mathbf{x}_0 = \mathbf{b}$.

4. Let A be a 3×3 matrix. Determine all possible reduced row echelon form of the augmented matrix $[A\ I_3]$ and decide whether A is invertible in each of the following cases.
(a) The matrix A has 1 pivoting column.
(b) The matrix A has 2 pivoting columns.
(c) The matrix A has 3 pivoting columns.

5. Suppose A is invertible, and $AB = AC$. Show that $B = C$.
[Hint: Show that $A(B - C) = 0$ ensures that $B - C = 0$ if A is invertible.]

6. Let $A = \begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -3 \end{bmatrix}$. Apply row reduction to the matrix $[\,A \quad I_3\,]$ until we have the reduced row echelon form $[\,I_3 \quad B\,]$, and verify that $AB = I_3$.

7. Let $A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$.
(a) Find a 4×2 matrix B such that $AB = I_2$.
[Hint: Solve $\mathbf{b_1}, \mathbf{b_2} \in \mathbf{R}^4$ such that $A\mathbf{b}_1 = \mathbf{e}_1 \in \mathbf{R}^2$ and $A\mathbf{b_2} = \mathbf{e}_2 \in \mathbf{R}^2$.
(b) Explain why there is no 4×2 matrix C such that $CA = I_4$.

8. Suppose A is an $n \times n$ matrix such that the columns of A^2 spans $\mathbb{R}^n$.
Show that A has linearly independent columns.

9. (10 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by

$$T(x_1, x_2) = (2x_1 - 8x_2, -2x_1 + 7x_2).$$

(a) Determine A such that $T(x_1, x_2) = (b_1, b_2)$ holds if $A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$, and compute A^{-1}.

(b) Define $R : \mathbb{R}^2 \to \mathbb{R}^2$ by $R(x_1, x_2) = (b_1, b_2)$ holds if $A^{-1} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$.

(c) Verify that $R \circ T(x_1, x_2) = (x_1, x_2)$ for all $x_1, x_2 \in \mathbb{R}$.

(d) Verify that $T \circ R(x_1, x_2) = (x_1, x_2)$ for all $x_1, x_2 \in \mathbb{R}$.