Math 211 Homework 5 LaTex file

1. If A, B, C are square matrices such that A and B are invertible such that $ABC = I$.
(a) Show that $BCA = I$.
(b) Give 2×2 examples showing that BAC may not be identity even if $ABC = I_2$.

2. Compute the product of two partitioned matrices:

$$\begin{bmatrix} I & 0 \\ -E & I \end{bmatrix} \begin{bmatrix} W & X \\ Y & Z \end{bmatrix}.$$

3. Suppose

$$\begin{bmatrix} A & B \\ 0 & I \end{bmatrix} \begin{bmatrix} X & Y & Z \\ 0 & 0 & I \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & I \end{bmatrix}.$$

Find formulas for X, Y, and Z in terms of A and B.

4. If A is $n \times n$ such that $A^2 = I_n$. Show that $M^2 = I_{2n}$ if $M = \begin{bmatrix} A & O_n \\ I_n & -A \end{bmatrix}$.

5. Let $A = \begin{bmatrix} A_{11} & O & O \\ O & A_{22} & O \\ O & O & A_{33} \end{bmatrix}$.

(a) If A_{11}, A_{22}, A_{33} are invertible matrices, show that $A^{-1} = \begin{bmatrix} A_{11}^{-1} & O & O \\ O & A_{22}^{-1} & O \\ O & O & A_{33}^{-1} \end{bmatrix}$.
[Here the O's in the partitioned matrix is the zero matrices of appropriate sizes.]
(b) Compute A^{-1} if $A_{11} = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ $A_{22} = \begin{bmatrix} 7 & 8 \\ 5 & 6 \end{bmatrix}$, $A_{33} = [2]$.
[Here A^{-1} should be a 5×5 matrix.]

6. We have the following LU factorization of the matrix

$$A = \begin{bmatrix} 2 & -6 & 4 \\ -4 & 8 & 0 \\ 0 & -4 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -6 & 4 \\ 0 & -4 & 8 \\ 0 & 0 & -2 \end{bmatrix}.$$

Solve the equation $A\mathbf{x} = \mathbf{b} = \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix}$ by first solving (a) $L\mathbf{y} = \mathbf{b}$, and then solving (b) $U\mathbf{x} = \mathbf{y}$.

7. Find the LU factorization of

$$A = \begin{bmatrix} -5 & 0 & 4 \\ 10 & 2 & -5 \\ 10 & 10 & 16 \end{bmatrix},$$

by applying elementary operations to yield $[A|I_n]$ to $[U|L^{-1}]$.

8. Suppose B is invertible and $A = BC$. Show that the reduced row echelon form of $[B\ A]$ is $[I\ C]$.
[Hint: We can find elementary matrices $E_1, \ldots, E_k$ such that $E_k \cdots E_1[B\ A] = [I\ C]$ according to the row operations applied to $[B\ A]$. Then, ...]