Five points for each question.

Math 211 Homework 6 Sample solution

1. Use cofactor expansion across the first row to find the determinant of

$$A = \begin{bmatrix} 8 & 1 & 6 \\ 4 & 0 & 3 \\ 3 & -2 & 5 \end{bmatrix}.$$

2. Use cofactor expansion to find the determinant of the matrix

$$A = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 7 & -1 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 5 & -8 & 4 & -3 \end{bmatrix}.$$

3. Use the Sarrus rule to find the determinant of the matrix

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{bmatrix}.$$

4. Use the row reduction algorithm to find the determinant of the matrix

$$A = \begin{bmatrix} 1 & 5 & -3 \\ 3 & -3 & 3 \\ 2 & 13 & -7 \end{bmatrix}.$$

5. Use the row reduction algorithm to find the determinant of the matrix

$$A = \begin{bmatrix} 1 & 3 & 3 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 5 & 4 & -3 \\ -3 & -7 & -5 & 2 \end{bmatrix}.$$

6. Suppose $\det\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right) = 7$. Deterimine $\det(M)$ and $\det(N)$ if

$$M = \begin{bmatrix} g & h & i \\ a & b & c \\ d & e & f \end{bmatrix} \text{ and } N = \begin{bmatrix} a+d & b+e & c+f \\ d & e & f \\ g & h & i \end{bmatrix}.$$

7. Let A be an $n \times n$ matrix.
(a) Show that $\det(rA) = r^n \det(A)$.
(b) Show that $\det(A^{-1}) = 1/\det(A)$ if $\det(A) \neq 0$.

8. Suppose A and P are $n \times n$ matrices such that P is invertible. Show that $\det(PAP^{-1}) = \det(A)$.

9. (10 points) Suppose A and B are 4×4 matrices, with $\det(A) = -1$ and $\det(B) = 2$.
Determine (a) $\det(AB)$, (b) $\det(B^5)$ (c) $\det(2A)$, (d) $\det(A^TA)$, (e) $\det(AB^{-1})$.