Math 211 Homework 7 Sample Texfile

1. Use Cramer's rule to compute the solution of the system

$$-5x_1 + 3x_2 = 9, \qquad 3x_1 - x_2 = -5.$$

2. Use Cramer's rule to compute the solution of the system.

$$2x_1 + x_2 + x_3 = 4, \qquad -x_1 + 3x_3 = 2, \qquad 3x_1 + x_2 + 3x_3 = 2.$$

3. Calculate the adjugate (adjoint) and the inverse of the following matrix:

$$A = \begin{bmatrix} 0 & -2 & -1 \\ 3 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix}.$$

4. Compute the area of parallelogram with the vertexes: $(0,-2), (6,-1), (-3,1), (3,2)$.
[Hint: Add $(0,2)$ to all the points to translate the parallelogram with $(0,0)$ as a vertex.]

5. Let

$$W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : xy \geq 0 \right\}.$$

1. Show that $c\mathbf{u} \in W$ for any $\mathbf{u} \in W$ and any $c \in \mathbb{R}$.
2. Find two vectors in W whose sum is not in W.

   [Hint: for the second part, you need to find $u = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$ such that $x_1y_1 \geq 0$ and $v = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$ so that $x_2y_2 \geq 0$. But $u + y$ have entries of different signs.]

6. Let W be all the vectors of the form $\begin{bmatrix} 2s+4t \\ 2s \\ 2s-3t \\ 5t \end{bmatrix}$. Show that W is a subspace of $\mathbb{R}^4$.

[Hint: Note that you need only show that the following three rules hold for W. (a) $0 \in W$, (b) $u+v \in W$ if $u, v \in W$, and (c) $cu \in W$ if $c \in \mathbf{R}$ and $u \in W$.]

7. Let $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : x_1 + x_2 + x_3 = 1 \right\}$. Show that W is not a vector space.

[Hint: You need to show that one of the three rules (a)–(c) fails in the previous hint.]

8. Let F be a fixed 3×2 matrix, and let $W = \{A : A \text{ is } 2 \times 4, FA = 0\}$. Show that W is a vector space.

[Hint: (a) Show that $A = O_{2\times 4}$ lies in W. (b) Show that if X, Y are 2×4 matrices in W, then $X + Y$ is a matrix in W. (c) Show that if $c \in \mathbf{R}$ and $X \in W$, then $cX \in W$.]

9. Let H and K be subspaces of a vector space V. Show that $H + K = \{x + y : x \in H, y \in K\}$ is a subspace.

[Hint: Note that H, K satisfy (a), (b), (c) rules in the Hints of Problem 6. Now, to show $H + K$ also satisfies the rules, one has to check:

(a) 0 can be written as $h + k$ with $h \in H, k \in K$ so that $0 \in H + K$.

(b) If $u = h_1 + k_1, v = h_2 + k_2 \in H + K$ where $h_1, h_2 \in H, k_1, k_2 \in K$, then $u + v$ can be written as $h + k$ with $h \in H$ and $k \in K$.

(c) If $c \in \mathbf{R}$ and $u = h_1 + k_1 \in H + K$ with $h_1 \in H, k_1 \in K$, then $cu = h + k$ for some $h \in H$ and $k \in K$.]