Math 211 Homework 10 Sample Texfile

1. (10 points) Show that

$$\mathbb{B} = \{1, 1-t, 2-4t+t^2, 6-18t+9t^2-t^3\}$$

is a basis for $\mathbb{P}_3$, and find the change of basis matrix from $\mathbb{B}$ to $\mathbb{C} = \{1, t, t^2, t^3\}$ and the change of the basis matrix from $\mathbb{C}$ to $\mathbb{B}$ Find $[u]_{\mathbb{B}}$ for $u = 1-3t+9t^2-t^3$.

2. (5 points) Find the rank, the nullity (dimension of null space), a basis for the column space, a basis for the row space of the following matrix

$$\begin{bmatrix} 1 & 1 & -2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & -3 & 0 \\ 0 & 0 & 1 & 1 & -13 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

3. Find the characteristic polynomial for the matrix and two linearly independent eigenvectors for the matrix

$$A = \begin{bmatrix} 4 & 4 \\ 1 & 4 \end{bmatrix}.$$

4. Show that $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ has at most one linear independent eigenvector.

5. Find the characteristic polynomial for the matrix and three linearly independent eigenvectors for the matrix

$$A = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 3 & 4 \\ 0 & 0 & 2 \end{bmatrix}.$$

6. Find the values h so that there are two linearly independent eigenvectors corresponding to the eigenvalue $\lambda = 4$ for the matrix

$$A = \begin{bmatrix} 4 & 2 & 3 & 3 \\ 0 & 2 & h & 3 \\ 0 & 0 & 4 & 14 \\ 0 & 0 & 0 & 2 \end{bmatrix}.$$

[Hint: Find h such that the null space of $A - 4I_4$ has nullity 2.]

7. Show that A and A^T have the same characteristic polynomial.
[Hint: Show that $\det(A - \lambda I) = \det(A^T - \lambda I)$.]