1. Given that $\lambda = 2, 3$ are eigenvalues for the matrix

$$A = \begin{bmatrix} 2 & 0 & -2 \\ 1 & 3 & 2 \\ 0 & 0 & 3 \end{bmatrix}.$$

(a) Find an invertible matrix P and a diagonal matrix D such that $A = P^{-1}DP$.

(b) Show that $(D - 2I)(D - 3I) = 0$ and $(A - 2I)(A - 3I) = 0$.

2. GIven a 7×7 matrix A has three different eigenvalues $\lambda_1, \lambda_2, \lambda_3$.
Suppose the $\mathrm{Nul}(A - \lambda_1 I)$ is three-dimensional and $\mathrm{Nul}(A - \lambda_2 I)$ is two-dimensional.

(a) If A is diagonalizable, what can we say about the dimension of $\mathrm{Nul}(A - \lambda_3 I)$?

(b) If A is not diagonalizable, what can we say about the dimension of $\mathrm{Nul}(A - \lambda_3 I)$?

3. Let $\mathcal{D} = \{\mathbf{d}_1, \mathbf{d}_2\}$ and $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ be the bases for vector spaces W and V respectively. Let $T : W \to V$ with

$$T(\mathbf{d}_1) = 3\mathbf{b}_1 - 3\mathbf{b}_2 \textit{ and } T(\mathbf{d}_2) = -2\mathbf{b}_1 + 5\mathbf{b}_2.$$

Find the matrix for T relative to $\mathcal{D}$ and $\mathcal{B}$.

4. Let $T : \mathbb{P}_2 \to \mathbb{P}_4$ be such that $T(\mathbf{p}(t)) = \mathbf{p}(t) + 2t^2\mathbf{p}(t)$.

(a) Find the image of $\mathbf{p}(t) = 3 - 2t + t^2$.

(b) Show that T is a linear transformation.

(c) Find a matrix for T relative to the bases $\{1, t, t^2\}$ and $\{1, t, t^2, t^3, t^4\}$.

5. Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ be a basis for a vector space V. Suppose $T : V \to V$ is a linear transformation whose matrix relative to the basis $\mathcal{B}$ is

$$[T]_{\mathcal{B}} = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & -2 \\ 1 & 3 & 1 \end{bmatrix}.$$

Find $T(4\mathbf{b}_1 - 3\mathbf{b}_2)$.

6. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(\mathbf{x}) = A\mathbf{x}$, where

$$A = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}.$$

Find a basis $\mathcal{B}$ with the property that $[T]_{\mathcal{B}}$ is diagonal.

7. Let $T : \mathbb{P}_1(t) \to \mathbb{P}_1(t)$ be defined by $T(a + bt) = (2a + 3b) + (3a + 2b)t$.

Find a basis $\mathcal{B}$ of $\mathbb{P}_1(t)$ with the property that $[T]_{\mathcal{B}}$ is diagonal.