1. Compute $\left(\frac{\mathbf{x}\cdot\mathbf{w}}{\mathbf{x}\cdot\mathbf{x}}\right)\mathbf{x}$, where

$$\mathbf{x} = \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix} \quad \text{and} \quad \mathbf{w} = \begin{bmatrix} 3 \\ -1 \\ -5 \end{bmatrix}.$$

2. Find a unit vector in the direction of the vector $\mathbf{x} = \begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix}$.

3. Let $\mathbf{u} = \begin{bmatrix} 12 \\ 3 \\ -5 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix}$, and $\mathbf{3} = \begin{bmatrix} 3 \\ -3 \\ 1 \end{bmatrix}$.
Check which pairs of the above vectors are orthogonal.

4. Consider the set of vectors

$$\left\{ \begin{bmatrix} 2 \\ -5 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}.$$

(a) Show that it is an orthogonal set.
(b) Find a nonzero vector orthogonal to the span of the set.

5. Show that $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ is an orthogonal basis for $\mathbb{R}^3$ if

$$\mathbf{u_1} = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}, \quad \mathbf{u_2} = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \quad \mathbf{u_3} = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}.$$

Express $\mathbf{x} = \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$ as a linear combination of $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$.

6. Let $\mathbf{y} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$. Express $\mathbf{y} = a\mathbf{u} + b\mathbf{z}$ such that $\mathbf{z}$ is orthogonal to $\mathbf{u}$.

7. Let $\mathbf{y} = \begin{bmatrix} -3 \\ 9 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Find the distance from $\mathbf{y}$ to the line passing through $\mathbf{u}$.

8. Let $\mathbf{x}_1 = \begin{bmatrix} -2/3 \\ 1/3 \\ 2/3 \end{bmatrix}$ and $\mathbf{x}_2 = \begin{bmatrix} 1/3 \\ h \\ 0 \end{bmatrix}$.
Determine h so that the two vectors are orthogonal;
then find positive numbers a, b so that $a\mathbf{x}_1$ and $b\mathbf{x}_2$ are orthonormal.