Math 211 Homework 13 Sample Texfile

10 points for each question.

1. Let

$$\mathbf{y} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}, \quad \mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}.$$

(a) Show that $\{\mathbf{u}_1, \mathbf{u}_2\}$ is an orthogonal set.
(b) Express $\mathbf{y} = \hat{\mathbf{y}} + \mathbf{z}$ so that $\hat{\mathbf{y}}$ is orthogonal projection of the vector $\mathbf{y}$ onto $W = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2\}$.
(c) Find the distance of $\mathbf{y}$ to the subspace W.
(d) Normalize the vectors in $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{z}\}$ to get orthonormal basis.

2. Let

$$\mathbf{u}_1 = \begin{bmatrix} 3 \\ 4 \\ 0 \\ 1 \end{bmatrix} \quad \text{and } \mathbf{u}_2 = \begin{bmatrix} -4 \\ 3 \\ 1 \\ 0 \end{bmatrix}.$$

(a) Show that $\{\mathbf{u}_1, \mathbf{u}_2\}$ is an orthogonal set.
(b) Express $\mathbf{y} = \hat{\mathbf{y}} + \mathbf{z}$ so that $\hat{\mathbf{y}}$ is orthogonal projection of the vector $\mathbf{y} = \begin{bmatrix} 6 \\ 3 \\ -2 \\ 2 \end{bmatrix}$ onto $W = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2\}$.
(c) Find the distance of $\mathbf{y}$ to the subspace W.
(d) Find a nonzero vector $\mathbf{z}_2$ so that $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{z}, \mathbf{z}_2\}$ is an orthogonal basis for $\mathbf{R}^4$.
(e) Normalize the vectors in S to get an orthonormal basis.

3. Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ 14 \\ -7 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}.$$

(a) Show that $\mathcal{B}$ is a basis for $\mathbf{R}^3$.
(b) Apply the Gram-Schmidt process to $\mathcal{B}$ to get an orthogonal basis $\mathcal{B}_1$.
(c) Normalize the vectors in $\mathcal{B}_1$ to get and orthnormal basis $\mathcal{B}_2$.
(d) Let A be the matrix with the vectors in $\mathcal{B}$ as columns, and let V be the matrix with the vectors in $\mathcal{B}_2$ as columns. Show that $V^T A = R$ is in upper triangular from.

4. Let

$$A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & -3 & 1 \\ 0 & 2 & 3 \\ 1 & 5 & 2 \\ 1 & 5 & 8 \end{bmatrix}.$$

(a) Show that A has linearly independent columns.
(b) Apply the Gram-Schmidt process to the columns of A to get an orthogonal basis $\mathcal{B}$ for the column space of A.
(c) Normalize the vectors in $\mathcal{B}$ to get an orthonormal basis $\mathcal{U}$ for the column basis for A.
(d) Let Q be the matrix with the vectors in $\mathcal{U}$ as columns. Show that $Q^T A = R$ is in upper triangular form.
Remark: We have $A = QR$, which is known as the QR factorization of A.

5. Repeat the procedures in Question 4 for the matrix $A = \begin{bmatrix} -2 & 3 \\ 5 & 7 \\ 2 & -2 \\ 4 & 6 \end{bmatrix}$ to find the QR factorization of A.