1. Note that $A^TA = \begin{bmatrix} 3 & 3 \\ 3 & 11 \end{bmatrix}$ and $A^Tb = \begin{bmatrix} 6 \\ 14 \end{bmatrix}$. Then we solve the linear system

$\begin{bmatrix} 3 & 3 & 6 \\ 3 & 11 & 14 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Thus, the least square solution is $\hat{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

2. We have

$$\hat{b} = \frac{b \cdot a_1}{a_1 \cdot a_1} a_1 + \frac{b \cdot a_2}{a_2 \cdot a_2} a_2 + \frac{b \cdot a_3}{a_3 \cdot a_3} a_3 = \begin{bmatrix} 5 \\ 2 \\ 3 \\ 6 \end{bmatrix}.$$

The least square solution is: $\begin{bmatrix} 1/3 \\ 14/3 \\ -5/3 \end{bmatrix}$.

3. Note that $Q^Tb = \begin{bmatrix} 17/2 \\ 9/2 \end{bmatrix}$. Then we solve the system $Rx = Q^Tb$:
$\begin{bmatrix} 2 & 3 & 17/2 \\ 0 & 5 & 9/2 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 29/2 \\ 0 & 1 & 9/10 \end{bmatrix}$. Thus, $\hat{x} = \begin{bmatrix} 29/10 \\ 9/10 \end{bmatrix}$.

4. Find an orthogonal basis for $\mathrm{Col}(A)^\perp$ for the matrix A in Problem 3.
Solution. Note that $\mathrm{Col}(A)^\perp = \{x \in \mathbf{R}^4 : A^Tx = 0\} = \mathrm{Null}(A^T)$. Direct computation show that $\mathrm{Null}(A) = \mathrm{span}\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$. The two vectors in the spanning set are orthogonal. So, the two vectors form an orthogonal basis.

5. Find the equation $y = a_0 + a_1x$ of the least square line that best fits the data: $(2,3), (3,2), (5,1), (6,0)$.
Solution. Assume $y = a_0 + a_1x$ is the least square line fit of the given data. Let

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 1 & 5 \\ 1 & 6 \end{bmatrix}, \quad \text{and} \quad y = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix}.$$

We need to solve $A^TA \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = A^tb$. Thus, we solve

$$\begin{bmatrix} 4 & 16 \\ 16 & 74 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 16 \\ 17 \end{bmatrix}$$

and get $(a_0, a_1) = (4.3, -0.7)$. Thus, the equation of the line equals: $y = 4.3 - 0.7x$