Math 214 Final Examination **Your Name:** ____________________________

Ten points for each question except for Problem 1. Good Luck!

(1) (Five points for each part.)

(a) Write the negation of the following statement:
"For any $a \geq 0$, there exists $n \in \mathbb{N}$ such that $1/n < a$."

(b) Prove or disprove the following. There is $n \in \mathbb{Z}$ such that $n^2 = 7k + 3$ for some $k \in \mathbb{Z}$.

(c) Prove or disprove the following. If $A \subseteq B$, and if B is denumerable, then A is denumerable.

(d) Consider the equivalence relation R on $\mathbb{R}^2$ defined by $((x_1, y_1), (x_2, y_2)) \in R$ if $x_1^2 + y_1^2 = x_2^2 + y_2^2$
Describe geometrically the equivalence class for $(1, 0)$.

(2) Use the Euclidean Algorithm to find $d = \gcd(346, 123)$, and find TWO pairs of points $(x, y) \in \mathbb{Z} \times \mathbb{Z}$ so that $2d = 346x + 123y$.

(3) Prove the following. (a) $\sqrt{2016}$ is irrational. (b) $\log(2016)$ is irrational.

(4) Suppose $a, b \in \mathbb{N}$ are distinct odd numbers and none of them is divisible by 3. Show that $a^2 - b^2$ is divisible by 24.

(5) Let $a_0 = 0$, $a_1 = 1$, and $a_n = 2a_{n-1} - a_{n-2} + 2$ for $n \geq 2$. Make a conjecture on the general form of a_n in terms of n and prove your conjecture using induction.

(6) Let R_1 and R_2 be two relations on a non-empty set A. Prove or disprove the following.

(a) If $R_1 \cup R_2$ is reflexive, then so are R_1 and R_2.

(b) If $R_1 \cup R_2$ is symmetric, then so are R_1 and R_2.

(c) If $R_1 \cup R_2$ is transitive, then so are R_1 and R_2.

(7) Let S and T be two sets. Prove that if $|S - T| = |T - S|$, then $|S| = |T|$.

(8) Show that the set of functions from $\{1, 2, 3\}$ to $\mathbb{N}$ is denumerable.

[You may use the facts: (a) $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$ is denumerable, (b) a subset of a denumerable set is finite or denumerable.]

(9) For nonempty sets A and B and functions $f : A \to B$ and $g : B \to A$, suppose that $f \circ g = 1_B$, the identity function on B. Prove that f is injective if and only if g is surjective.

(10) Let $\mathcal{P}(A)$ be the power set of a non-empty set A. Show that $|A| < |\mathcal{P}(A)|$.

Question	1	2	3	4	5	6	7	8	9	10	Total
Score											