Math 214 – Foundations of Mathematics **Homework 1** **Your name**

Solve the following problems. Six points for each problem.

1. Write each of the following sets as specified.
   (a) List the elements in the set $A = \{n \in \mathbf{N} : n^3 < 1000\}$.
   (b) Describe the set $B = \{-2, -1, 0, 1, 2, 3, 4\}$ using the notation $B = \{n : p(n)\}$, where $p(n)$ specifies the property of element n.
2. Recall that for a set A, $\mathcal{P}(A)$ denotes the power set of A.
   (a) Find $\mathcal{P}(\mathcal{P}(\{a\}))$ and its cardinality.
   (b) Give an example of a set S such that $S \in \mathcal{P}(\mathbf{N})$ and $|S| = 6$.
   (c) Give an example of a set S such that $S \subseteq \mathcal{P}(\mathbf{N})$ and $|S| = 6$.
3. The following problems involve set operations.
   (a) Give an example of three non-empty sets A, B, and C such that $B \neq C$ but $B - A = C - A$.
   (b) Let $A = \{\emptyset, \{\emptyset\}\}$. Find $\mathcal{P}(A) - A$.
4. For a real number r, define S_r to be the interval $[r-1, r+2)$. Let $A = \{1, 3, 4\}$.

(a) List the intervals S_r for $r \in A$.

(b) Determine $\cup_{\alpha \in A} S_\alpha$ and $\cap_{\alpha \in A} S_\alpha$.

(c) (Extra 3 points) Determine $\cap_{r \in (0,1)} S_r$ and $\cup_{r \in (0,1)} S_r$.

5. For two sets A and B, let $A \times B = \{(a, b) : a \in A, b \in B\}$ be their Cartesian product.
   (a) Let $A = \{a, b\}$. Determine $A \times \mathcal{P}(A)$.
   (b) Let $A = \{0, 1\}$ and $B = [0, 2] \cap [1, 3]$. Describe geometrically the set $A \times B$ in $\mathbf{R}^2 = \mathbf{R} \times \mathbf{R}$.
   (c) Let $A = \{0, 1\}$, $B = (0, 1) \cap A$ and $C = \mathbf{R}$. What is $A \times B \times C$?
6. Determine all different partitions of the set $\{1, 2, 3\}$.

Extra credit problem. (3 points) If a set A has n elements, show that $\mathcal{P}(A)$ has 2^n elements.