Math 214 Homework 2 **Your name**

1. (4 points) Let P: 25 is odd, Q: 15 is prime and R: $\frac{1}{3} \in \mathbb{N}$. State each of the following in words, and determine whether they are true or false.

   (a) $P \vee Q$

   (b) $P \wedge Q$

   (c) $(\sim P) \vee Q$

   (d) $P \wedge (\sim Q \vee R)$

2. (4 points) In each of the following, two open sentences $P(x)$ and $Q(x)$ over a domain S are given. For each part, determine $T = \{x \in S : \text{"}P(x) \Rightarrow Q(x)\text{" is true}\}$ with explanation.

   (a) $P(x) : x - 3 = 4; Q(x) : x \geq 8; S = \mathbb{R}$.

   (b) $P(x) : x \in [-1, 2]; Q(x) : x^2 \leq 2; S = [-1, 1]$.

3. (4 points) Let P, Q be statements. Show that $\sim (P \implies Q)$ and $P \wedge (\sim Q)$ are logically equivalent using truth table.

   Solution.

P	Q	$P \implies Q$	$\sim (P \implies Q)$	$P \wedge (\sim Q)$
T	T			
T	F			
F	T			
F	F			

4. (6 points) Write the statements so that there are no $\sim$ symbols. Then, rewrite the statements so that there are no $\forall, \exists, \in$ or $=$ symbols.

   (a) $\sim (\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, xy = 1)$;

   (b) $\sim (\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, xy = 0)$;

   (c) $\sim (\exists n \in \mathbb{Z}, \exists m \in \mathbb{Z}, m \leq n)$;

5. (4 points) Consider the statement:

   "For every integer $n > 0$ there is some real number $x > 0$ such that $x < 1/n$."

   (a) Without using words of negation, write a complete sentence that negates the sentence.

   (b) Determine the original statement or its negation is true with explanation.

6. (8 points) For $\alpha > 0$, let $S_\alpha = (-\alpha, \alpha)$, i.e., the open interval with endpoints $-\alpha, \alpha$. Prove or disprove the following statements.

   (a) $\forall \alpha \in (0, 1), \exists \beta \in (0, 1), S_\alpha \subset S_\beta$ (note that $\subset$ and $\subseteq$ are not the same).
   [Hint: Try to find β if $\alpha = 0.9, 0.99, 0.999$, etc. and find the general rule for specifying β for a given α.

   (b) $\exists \alpha \in (0, 1), \forall \beta \in (0, 1), S_\alpha \subset S_\beta$.
   [Hint: For each of $\alpha = 0.1, 0.01, 0.001$, see whether we can say that $S_\alpha \subset S_\beta$ for all $\beta \in (0, 1)$. Then ...]