Math 214 Homework 7 **Your name**

1. (6 points) Solve the following problems in $\mathbf{Z}_n$.

   (a) In $\mathbf{Z}_8$, express the following sums and products as $[r]$, where $0 \leq r < 8$:

   $$[3] + [6], \quad [3][6], \quad [-13] + [138], \quad [-13][138].$$

   (b) Let $[a], [b] \in \mathbf{Z}_8$. If $[a][b] = [0]$, does it follow that $[a] = [0]$ or $[b] = [0]$

   (c) Prove that for any prime p, if $[a], [b] \in \mathbf{Z}_p$, then $[a][b] = [0]$ implies $[a] = [0]$ or $[b] = [0]$.

   [Hint: $[ab] = [0]$ in $\mathbf{Z}_n$ means ab is divisible by n.]

2. (4 points) A relation R is defined on $\mathbf{Z}$ by $(a, b) \in R$ if $|a - b| \leq 2$. Which of the properties reflexive, symmetric, and transitive does the relation R possess? Justify your answers.

3. (4 points) Let R be a relation defined on $\mathbf{Z} - \{0\}$ by $(a, b) \in R$ if $ab > 0$. Show that R is an equivalence relation on $\mathbf{Z} - \{0\}$.

4. (8 points) Find relations on $S = \{1, 2, 3\}$ satisfying the following. Verify your answers.

   (a) Reflexive, symmetric, not transitive.

   (b) Reflexive, not symmetric, not transitive.

   (c) Symmetric, transitive, not reflexive,

   (d) Symmetric, not reflexive, not transitive.

5. (8 points) Find relations on $\mathbf{Z}$ satisfying the following. Verify your answers.

   (a) Reflexive, symmetric, not transitive.

   (b) Reflexive, not symmetric, not transitive.

   (c) Symmetric, transitive, not reflexive,

   (d) Symmetric, not reflexive, not transitive.

6. (a) (3 points) Define the relation R on $\mathbf{R}^2$ by $(x_1, y_1)R(x_2, y_2)$ if $|x_1| + |y_1| = |x_2| + |y_2|$. Prove that R is an equivalence relation, and describe the geometrical shape of the disjoint equivalence classes of R in $\mathbf{R}^2$.

   (b) (3 points) Consider the partition of $\mathbf{R}^2$ by straight lines $L_r = \{(x, y) : x + y = r\}$ for each $r \in \mathbf{R}$. Show that $P = \{L_r : r \in \mathbf{R}\}$ is a partition of $\mathbf{R}^2$, and define an equivalence relation so that L_r's are the equivalence classes.

   [Remark: We expect to see an answer saying that: $(x_1, y_1)R(x_2, y_2)$ if ....]

7. (6 points) Determine with explanation all the equivalence relations on $S = \{1, 2, 3\}$.