## Math 214 Homework 7

## Your name

- 1. (6 points) Solve the following problems in  $\mathbf{Z}_n$ .
  - (a) In  $\mathbb{Z}_8$ , express the following sums and products as [r], where  $0 \le r < 8$ :

[3] + [6], [3][6], [-13] + [138], [-13][138].

- (b) Let  $[a], [b] \in \mathbb{Z}_8$ . If [a][b] = [0], does it follow that [a] = [0] or [b] = [0]
- (c) Prove that for any prime p, if  $[a], [b] \in \mathbb{Z}_p$ , then [a][b] = [0] implies [a] = [0] or [b] = [0].

[Hint: [ab] = [0] in  $\mathbf{Z}_n$  means ab is divisible by n.]

- 2. (a) (5 points) A relation R is defined on Z by (a, b) ∈ R if |a b| ≤ 2. Which of the properties reflexive, symmetric, and transitive does the relation R possess? Justify your answers.
  (b) (5 points) Let R be a relation defined on Z {0} by (a, b) ∈ R if ab > 0. Show that R is
  - (b) (5 points) Let R be a relation defined on  $\mathbb{Z} \{0\}$  by  $(a, b) \in R$  if ab > 0. Show that R is an equivalence relation on  $\mathbb{Z} \{0\}$ .
- 3. (8 points) Find relations on  $S = \{1, 2, 3\}$  satisfying the following. Verify your answers.
  - (a) Reflexive, symmetric, not transitive.
  - (b) Reflexive, not symmetric, not transitive.
  - (c) Symmetric, transitive, not reflexive,
  - (d) Symmetric, not reflexive, not transitive.
- 4. (8 points) Find relations on Z satisfying the following. Verify your answers.
  - (a) Reflexive, symmetric, not transitive.
  - (b) Reflexive, not symmetric, not transitive.
  - (c) Symmetric, transitive, not reflexive,
  - (d) Symmetric, not reflexive, not transitive.
- 5. (a) (6 points) Define the relation R on  $\mathbf{R}^2$  by  $(x_1, y_1)R(x_2, y_2)$  if  $|x_1| + |y_1| = |x_2| + |y_2|$ . Prove that R is an equivalence relation, and describe the geometrical shape of the disjoint equivalence classes of R in  $\mathbf{R}^2$ .

(b) (6 points) Consider the partition of  $\mathbf{R}^2$  by straight lines  $L_r = \{(x, y) : x + y = r\}$  for each  $r \in \mathbf{R}$ . Show that  $P = \{L_r : r \in \mathbf{R}\}$  is a partition of  $\mathbf{R}^2$ , and define an equivalence relation so that  $L_r$ 's are the equivalence classes.

[Remark: We expect to see an answer saying that:  $(x_1, y_1)R(x_2, y_2)$  if ....]

6. (6 points) Determine with explanation all the equivalence relations on  $S = \{1, 2, 3\}$ . [Hint: Consider all partitions of S.]