

## Math 214 Foundations of Higher Mathematics

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Meeting time and place: TT 09:00-11:50. Jones Hall 306

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李志光

### Course description

The goal of the course is to provide the training for the mathematical background to study advanced courses, which require abstract thinking and to understand and write proofs for mathematical statements.

The course will cover the first 12 chapters (Chapter 0 - 11, yes, starting from 0!) of the following required textbook.

G. Chartrand, A.D. Polimeni, P. Zhang, *Mathematical Proofs, A transition to advanced mathematics*, Newest Edition. Addison Wesley, New York, 2008.

### Homework

There is a weekly homework set due on Friday noon starting January 27.

See the homework list on the course website <http://cklix.people.wm.edu/teaching/m214.html>

Homework sessions will be conducted on Wednesday, 3:00-4:00 p.m. at Jones 113 or Jones 131.

Extra credit problems will be assigned from time to time.

Extra-credits will be given to successful (or partially successful) attempts.

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### LaTeX

You have to use LaTeX to typeset mathematical document, an excellent skill to acquire!

- For windows, you may
  - (a) download the MikTeX program from <http://miktex.org/download>;
  - (b) then download the Texmaker program from <http://www.xmlmath.net/texmaker/download.html>.
- For Mac users, you may download MacTeX from <http://tug.org/mactex/>.
- You may also use the online editor [Write LaTeX https://www.writelatex.com/](https://www.writelatex.com/).
- Here is a list of TeX commands for mathematics symbols <http://cklix.people.wm.edu/TeX-symbol.pdf>.

### Assessment

Quizzes (20 min. each) Feb. 2, 16, March 23, April 6, 20.

Exams: Mid-term March 2 1:20 hrs (09:00-11:50)

Final May 8 3 hrs (09:00-12:00)

Grades (for homework, quizzes, exams, final grade, etc.):

#: 0 - 60 - 65 - 70 - 75 - 80 - 83 - 87 - 90 - 93 - 100

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|             |          |         |          |       |
|-------------|----------|---------|----------|-------|
| Assessment: | Homework | Quizzes | Mid-term | Final |
|             | 20%      | 20%     | 25%      | 35%   |

(Extra credit problems may add up to another 5%)

## About this course

- It is very different from other computational courses.
- It is like learning a new (mathematical) language.
- You have to understand and MEMORIZE the definitions, theorems, proofs, etc.
- You may ask me to show you examples, but better create your own for each new concept.
- I assume that you are familiar with basic manipulations of numbers, functions, and calculus.
- I also assume that you will read the textbook carefully, say, Chapters 0 & 1 in this weekend!

## Class notes

Notes including the discussion in class will be scanned and put on the course webpage for your reference. They might help you to review the techniques introduced in class, and the explanation of the ideas behind the definitions, theorems, and proofs.

The organization of the notes will be different from that in the textbook, and cannot be used as a quick references for quizzes and examinations if you do not come to classes or read the textbooks.

Chapter 1 Sets

**Definition** A set is a collection of objects, its elements (or members).

**Notation**

We use curly bracket  $\{ \}$  to embrace the elements in the set, elements are separated by comma ',' in the brackets,

$x \in S$ :  $x$  is an element (member) of  $S$ , or  $x$  belongs to the set  $S$ ;

$x \notin S$ :  $x$  is not an element (member) of  $S$ ,  $x$  does not belong to the set  $S$ .

$|S|$ : Cardinality (size, or cardinal number) of a set  $S$ , which may be finite or infinite.

A set is finite if  $|S| = n$  for some nonnegative integer  $n$ .

$\{x \mid x \text{ is a real number satisfying } x^2 = 4\}$

**1.1 Describing a set** by (a) listing all elements; (b) describing the property.

$A = \{a, e, i, o, u\}$ ;  $A$  is the set of vowels;  $a \in A$ ;  $a, e \in A$ ,  $m \notin A$ ;  $|A| = 5$ .

$B = \{\text{cat, dog, pig}\}$  is the set consisting of the elements: cat, dog, pig;  $|B| = 3$ .

$C = \{1, 2, \{1, 3\}, \text{cat}\}$ ;  $\{1, 3\} \in C$ ;  $\text{cat} \in C$ ;  $1 \in C$ ;  $3 \notin C$ .

$D = \{-2, 2\} = \{x \in \mathbb{R} \mid x \text{ is a real number such that } x^2 = 4\}$   
 $= \{x \mid x \text{ is a real number such that } |x| = 2\}$ .

$E = \{1, 3, 5, 7\} = \{x \mid (x-1)(x-3)(x-5)(x-7) = 0\}$ ;  
 $E$  is the set of odd integers between 0 and 8.

**Remark** Two sets  $A$  and  $B$  are equal, denoted by  $A = B$ , if they contain the same elements.

That is, every element in  $A$  is an element in  $B$ , and vice versa.

**Example**  $\{1, 2, 3\} = \{1, 3, 2\} = \{1, 2, 2, 3\} = 3$ .

multi-sets may ~~can~~ have elements with multiplicities more than 1.

Special sets = { } { }

Empty set  $\emptyset$ . It is also called the null set or the void set. Note that  $|\emptyset| = 0$ .

Example Suppose  $F = \{\emptyset, \{\emptyset\}\}$ .  $\mathbb{E} \rightarrow 2$   
(a)  $\emptyset \in F?$  (b)  $\{\emptyset\} \in F?$  (c)  $|F| = ???$

We always assume that there is a universal set  $U$  containing all the objects under consideration.

It will lead to a (Russell) paradox if we assume that there is a set containing EVERYTHING.

$\mathbb{N} = \{1, 2, 3, 4, \dots\}$  is the set of natural numbers.

$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ .

$\mathbb{Q} = \{m/n : m \in \mathbb{Z}, n \in \mathbb{N}\} = \{a/b : a, b \in \mathbb{Z}, b \neq 0\}$  is the set of rational numbers.

$\mathbb{R}$  is the set of real numbers.

$\mathbb{C} = \{a + ib : a, b \in \mathbb{R}\}$  is the set of complex numbers.

Remark All of the above sets are infinite, say,  $|\mathbb{N}|$  is infinite, or we write  $|\mathbb{N}| = \infty$ .