Special sets $\{\ \}$

Empty set $\emptyset$. It is also called the null set or the void set. Note that $|\emptyset| = 0$.

Example Suppose $F = \{\emptyset, \{\emptyset\}\}$. $\quad |F| \rightarrow 2$

(a) $\emptyset \in F$? $\qquad$ (b) $\{\emptyset\} \in F$? $\qquad$ (c) $|F| =$???

We always assume that there is a $\boxed{\textbf{universal} \text{ set } U}$ containing all the objects under consideration.

It will lead to a (Russell) paradox if we assume that there is a set containing EVERYTHING.

$3+x=5$

$\mathbb{N} = \{1, 2, 3, 4, \dots\}$ is the set of natural numbers. $\quad$ Use Peano's axiom to construct $\mathbb{N}$ $\{0, 1, 2, \dots\}$

$\boxed{x+1=0}$ $\to$ $\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$.

$3x=1$ $\quad$ $\mathbb{Q} = \{m/n : m \in \mathbb{Z}, n \in \mathbb{N}\} = \{a/b : a, b \in \mathbb{Z}, b \neq 0\}$ is the set of rational numbers. $\quad \in \mathbb{R}$

$\mathbb{R}$ is the set of real numbers. $\quad -1 \quad 0 \quad 1 \quad 2 \quad 3$ $\quad$ Math311

$x^2=-1$ $\quad$ $\mathbb{C} = \{a + ib : a, b \in \mathbb{R}\}$ is the set of complex numbers.

Remark All of the above sets are infinite, say, $|\mathbb{N}|$ is infinite, or we write $|\mathbb{N}| = \infty$.

Let $S = \{2, 4, 6, 8, \dots\}$. be the set of positive even integers

Fact: If $|X| = n$, $n \in \{0, 1, 2, 3, \dots\}$.
then $|\mathcal{P}(X)| = 2^n$.

(Try to explain it clearly in your X-credit homework)

Example $F = \{\phi, \{\phi\}\}$ $|F| = 2$

$|\mathcal{P}(F)| = 4$.

$\mathcal{P}(F) = \{\phi, F, \{\phi\}, \{\{\phi\}\}\}$

1.2 Subsets Let X, Y be sets.

\subseteq

Definition A set X is a subset of Y if every element in X is an element in Y, denoted by $X \subseteq Y$.

If in addition that $X \neq Y$, then X is a proper subset of Y.

$X \subset Y$ \subset

$X \subsetneq Y$

Example $X = \{1,2,3\}, Y = \{1,2,3,4,5\}, Z = \{4\}$.

$X \subseteq Y$, $Z \subseteq Y$, $Z \not\subseteq X$ \not\subseteq

$X \subsetneq Y$, $Z \subsetneq Y$

Example $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$.

Remark Two sets A and B are equal if $A \subseteq B$ and $B \subseteq A$.

Definition The set of all subsets of X is the power set of X, denoted by $\mathcal{P}(X)$. \P

Example (a) $X = \emptyset$. (b) $Y = \{1\}$. (c) $Z = \{0, \emptyset, \{\emptyset\}\}$.

Distinction between subsets and memberships

Example Suppose $F = \{\emptyset, \{\emptyset\}\}$.

e.g. $X = \{x \in \mathbb{R} : x^2 = 4\} = \{-2, 2\}$

$Y = \{1, 2, 3, -1, -2, -3\}$

Clearly $-2, 2 \in X$ we also have $-2, 2 \in Y$.

So $X \subseteq Y$.

1° Let X be a set. X has a subset, namely the empty set ϕ

So $\phi \subseteq X$

2° Let $X = \phi$. Then $\phi \subseteq \phi$.

Note $\phi \notin \phi$

Example (a) $X = \phi = \{ \}$. $\mathcal{P}(X) = \{ \phi \}$

$|X| = 0$ $|\mathcal{P}(X)| = 1$.

(b) $Y = \{1\}$ $\mathcal{P}(Y) = \{ \phi, \{1\} \} = \{\phi, Y\}$

$|Y| = 1$ $|\mathcal{P}(Y)| = 2$

(c) $Z = \{0, \phi, \{\phi\}\}$ ($\{0, \square, \triangle\}$), $\mathcal{P}(Z) = \{\phi, Z, \{0\}, \{\phi\}, \{\{\phi\}\}, \{0, \phi\}, \{0, \{\phi\}\}, \{\phi, \{\phi\}\}\}$

$|Z| = 3$ $|\mathcal{P}(Z)| = 8$

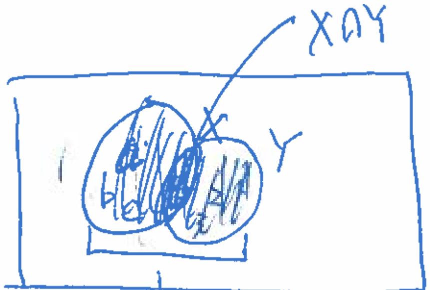
1.3 Set operations and Venn diagrams

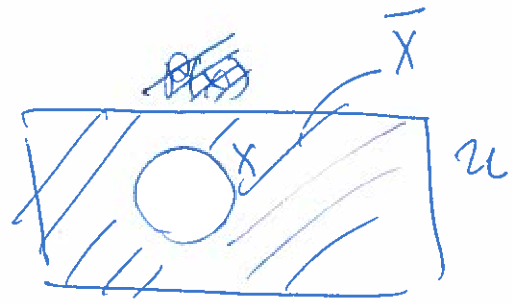
Venn diagrams can help depict the relationships and operations on sets.

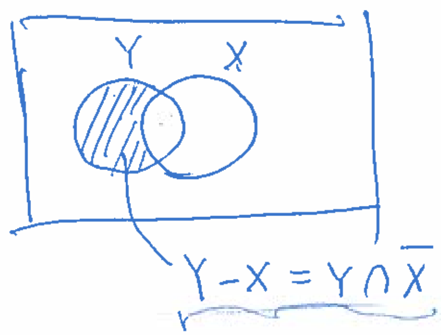
Definition Let X and Y be sets. in $\mathcal{U}$.

- Their union, denoted by $X \cup Y$, is the set $\{x \in U : x \in X \textbf{ or } x \in Y\}$.
- Their intersection, denoted by $X \cap Y$, is the set $\{x \in U : x \in X \textbf{ and } x \in Y\}$.
- The complement of X in the universal set U, denoted by $\overline{X}$, is the set $\{x \in U : x \notin X\}$.
- The relative complement of X in Y, denoted by $Y - X$, is the set $\{x \in U : x \in Y, x \notin X\}$.

Example $X = \{1, 2, 3\}, Y = \{1, 2, 3, 4, 5\}, Z = \{4\}$.







$$X = \{x \in \mathbb{C} : x^2 = -1\} = \{i, -i\}$$

$$Y = \{x \in \mathbb{R} : x^2 = -1\} = \phi$$