Math 214 QUIZ 3 Name: Sample Solution

1. It is known that $7 = \gcd(105, 154)$.

(a) Find a pair of integers (x, y) such that $105x + 154y = 7$.

(b) Find a pair of integers (u, v) such that $105u + 154v = 28$.

(a) $154 = 105 + 49$, $105 = 2 \times 49 + 7$

$\therefore 7 = 105 - 2 \times 49 = 105 - 2 \times (154 - 105) = (-2) \times (154) + (3) \times (105)$

$\therefore (x, y) = (3, -2)$ //

(b)

$28 = 7 \times 4$.

So we can let $(u, v) = (12, -8)$.

So that $(12)(105) + (-8)(154) = 28$.

2. Suppose c is a common divisor of $a, b \in \mathbf{Z}$. Show that c is a divisor of $ax + by$ for any $x, y \in \mathbf{Z}$.

If $c \mid a$ & $c \mid b$

then $a = cq_1$ and $b = cq_2$, $q_1, q_2 \in \mathbb{Z}$.

So for any $x, y \in \mathbb{Z}$.

$ax + by = cq_1 x + cq_2 y = c(\underline{q_1 x + q_2 y})$.

$\therefore c \mid (ax + by)$ for any $x, y \in \mathbb{Z}$.

Equivalence relation on a set S.

Set up: A set S.

A relation R on S i.e., a rule telling you where are two elements $a, b \in S$ such that a & b are related.

aRb, or $(a,b) \in R$

The relation R is reflexive if $(\forall a \in S)((a,a) \in R)$

The relation R is symmetric if $(\forall a,b \in S, (a,b) \in R)\ ((b,a) \in R)$

The relation R is transitive if $(\forall a,b,c \in S\ (a,b) \in R, (b,c) \in R)\ ((a,c) \in R)$.

The relation R is an equivalence relation if it is Reflexive, Symmetric, and Transitive

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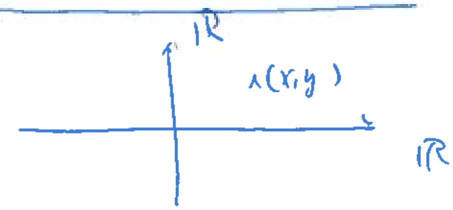
Example $S = \{(x,y) : x, y \in \mathbb{R}\}$



R: $(x_1,y_1) R (x_2,y_2)$ / $((x_1,y_1),(x_2,y_2)) \in R$

if $x_1^2 + y_1^2 = x_2^2 + y_2^2$.

$(1,0) R (0,1)$ ✓

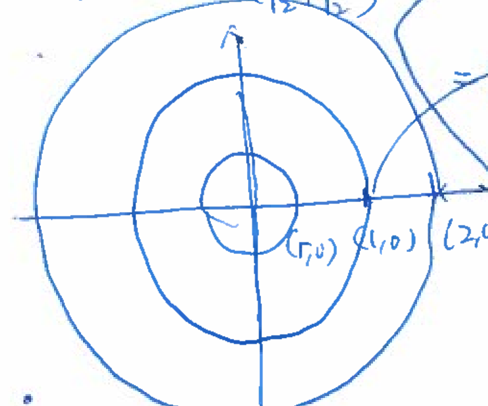
$(1,0) R (\frac{1}{2}, \frac{1}{2})$ ✗

$(1,0) R (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ ✓

Check

R: $\forall (x_1,y_1) \in S$, $(x_1,y_1) R (x_1,y_1)$ because $x_1^2+y_1^2 = x_1^2+y_1^2$.

$[(1,0)] = \{(x,y) \in S : x^2+y^2=1\}$



S: $\forall (x_1,y_1), (x_2,y_2) \in S$

such that $(x_1,y_1) R (x_2,y_2)$

i.e. $x_1^2+y_1^2 = x_2^2+y_2^2$

$\therefore$ $x_2^2+y_2^2 = x_1^2+y_1^2$

$\therefore$ $(x_2,y_2) R (x_1,y_1)$

T: $\forall (x_1,y_1), (x_2,y_2), (x_3,y_3) \in S$.

$(x_1,y_1) R (x_2,y_2)$, $(x_2,y_2) R (x_3,y_3)$.

i.e., $x_1^2+y_1^2 = x_2^2+y_2^2$, $x_2^2+y_2^2 = x_3^2+y_3^2$

$\therefore$ $x_1^2+y_1^2 = x_2^2+y_2^2 = x_3^2+y_3^2$.

<u>Example</u> $S = \{(x, y, z) : x, y, z \in \mathbb{R}\}$

$(x_1, y_1, z_1) \, R \, (x_2, y_2, z_2)$ if $x_1^2 + y_1^2 + z_1^2 = x_2^2 + y_2^2 + z_3^2$

Theorem: Let R be an equivalence relation on S.

For every $a \in S$, define the equivalence class of a by $[a] = \{ x \in S : aRx \}$.

Then ① For $a, b \in S$, $[a] = [b]$ or $[a] \cap [b] = \phi$.

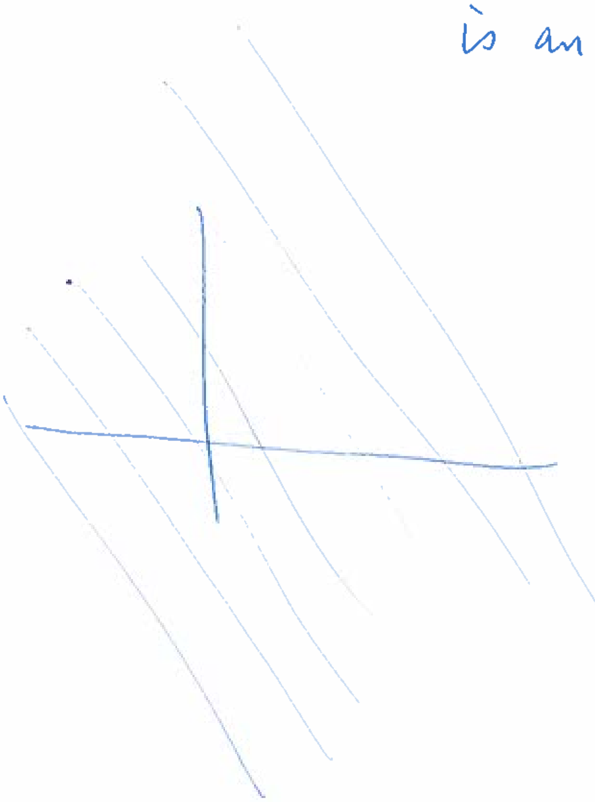
② $S = \bigcup_{a \in S} [a]$.

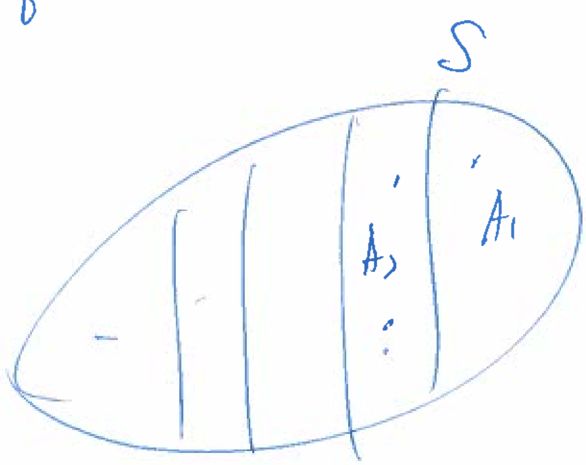
Theorem If S has a partition $\{ A_\lambda : \lambda \in \Lambda \}$

then R defined by

aRb if $a, b \in A_\lambda$ for some $\lambda \in \Lambda$

is an equivalence relation.





a b