Chapter 2 Logic

We study the mathematical language to read and write correct (logical) mathematics.

2.1 Statements

- A **statement** is a sentence/assertion which we can decide that it is true (T) or false (F).
- Examples. The integer 57 is a prime number. It is raining now. $2 + 4 = 6$.
- An **open sentence** is an assertion with one or more variables chosen from a domain S.
- Example. $P(x)$: $x > 3$. Here the domain can be $\mathbb{N}$, $\mathbb{Q}$, or $\mathbb{R}$.
- A statement may be true (T) or false (F); two statements have 4 possible combination;
- 3 statements have 8 possible combination; ...; n statements have 2^n combination.
- We may draw the truth table for that.

2.2/2.3 Negation, disjunction, and conjunction of statements

- Negation of P, denoted by $\sim P$;
- Disjunction: P or Q, denoted by $P \vee Q$;
- Conjunction: P and Q, denoted by $P \wedge Q$.
- Examples and truth tables.

2.4/2.5/2.6 Implication and biconditional

- Implication: If P then Q (also, P implies Q), denoted by $P \Rightarrow Q$.
- The statement P is the hypothesis/premise, and the statement Q is the conclusion.
- Biconditional: P is equivalent to Q (also, P if and only if Q), denoted by $P \iff Q$.
- Examples and truth tables.

2.7 Tautologies and contradiction

Tautology In a compound statement, all possible combination of the components yield T.

Contradiction In a compound statement, all possible combination of the components yield F.

2.8/2.9 Logical equivalence and properties

Two compound statements R and S are logically equivalent, denoted by $R \equiv S$ when they have the same truth values for different combination of the component statements.

Examples.

$(P \Rightarrow Q) \equiv (\sim P \vee Q)$;

Commutative, associative, and distributive laws for $\vee$ and $\wedge$;

De Morgan's Law.

2.10 Quantifiers

There exists: $\exists$; for all: $\forall$.

These usually go with open statements with variables from a domain.

Examples For every real number r, $(r+1)^2 > 0$. $\forall\, r \in \mathbb{R}, (r+1)^2 > 0$.

There is a real number r such that $r^2 = -1$. $\exists\, r \in \mathbb{R}, r^2 = -1$.

2.11 Characterizations

In mathematics, we often wants to give equivalent conditions for a certain concept. That is, give characterizations for a certain property or structure.