Q2:Compare other image compression methods and compare their advantages/disadvantages.

Fourier transform/DFT and Wavelet technique both use the change of basis and reduction of coefficients.

Some efficient bases:
$\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ (enough to give solid and plain image), $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$ (give image like checkboard)

Using these bases is good for simple image as it reduces the number of basis stored.

1. Fourier transform.

Use of Fourier basis.

Fourier transform(DFT) uses sine and cosine waves while DCT uses cosine waves.

JPEG uses fourier transform to compress image.

Step 1:

Divide 512 $\times$512 image into 8$\times$8 block.

Step 2(lossless step):

Change of basis to Fourier basis, like $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ : \end{pmatrix}$ and $\begin{pmatrix} 1 \\ w \\ w^2 \\ w^3 \\ w^4 \\ : \end{pmatrix}$ where w^n=1, w=$e^{i2\pi/n}$ in this case n=64.

Step 3(lossy step):

Reduce the number of coefficients. Usually small coefficients are deleted.

For example, a 8$\times$8 matrix block(64 coefficients) can be reduced to using only 30 coefficients. Extreme case, we can use only 1 coefficient to represent a single-color image. The amount of pixel stored is significantly reduced.

2. Wavelet transform:

The steps involved similar while wavelet transform uses wavelet basis.

For 8 dimensional space:

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

Advantages of fourier transform:

-Fast. "Fast fourier transform technique" makes the whole process quick.

-Lowest error for black and white images(Chen and Duan P.104).

Disadvantages of fourier transform:

-Always stop at some ratio and cannot compress further(Chen and Duan).

-Generally works better for black and white image than color images, due to accumulated layer error(Chen and Duan).

-Directional property(works well for parallel image but not curve image)(Chen and Duan).

Advantages of Wavelet transform:

-Fast. "Fast Wavelet transform technique" makes the whole process quick.

-Transition matrix can be easily inverted. It consists of orthogonal columns and the inverse is its transpose if the column is made orthnormal.

Disadvantages of Wavelet transform:

-Always stop at some ratio and cannot compress further(Chen and Duan).

-Generally works better for gray image than color images, due to 3-color accumulated layer error(Chen and Duan).

Advantages of SVD:

-Do not generally have different error ratios for gray and color images.

-Can compress image further than Fourier and Wavelet transform(Chen and Duan).

-Compress up to contour so general shape is retained(Chen and Duan P.103).

-Save largest amount of memory in general(Chen and Duan P.103)

-More stable(Chen and Duan P.104)

Disadvantages of SVD:

-Image quality is in some case not as good as Fourier and Wavelet.

Other SVD applications:

-Linear Dynamical system

References:

1. The Scientist and Engineer's Guide to Digital Signal Processing By Steven W. Smith, Ph.D.

Chapter 27: Data Compression

2. Gilbert strang video lecture Lec 26 MIT 18.06 Linear Algebra, Spring 2005.