**Math 307 – Abstract Algebra    Homework 1**

Solve the following problems. Five points for each question.

1. Let $a$ and $b$ be integers. Show that $a \mod n = b \mod n$ if and only if $a - b$ is divisible by $n$.

2. Show that $5n + 3$ and $7n + 4$ are relatively prime for all $n \in \mathbb{N}$.

3. Determine $7^{1000} \mod 6$, and $6^{1001} \mod 7$.

4. Prove that $2^n 3^{2n} - 1$ is always divisible by 17 for all non-negative integers $n$.

5. Let $\mathbb{R}$ be the set of real numbers. Define a relation $R$ on $\mathbb{R}$ by $(a, b) \in R$ if $a - b$ is an integer. Prove that $R$ is an equivalence relation and determine the equivalence classes.

6. Express the following complex numbers in standard form:

$$(-7 - 3i)^{-1}, \quad \frac{-5 + 2i}{4 - 5i}.$$

7. (Extra credit) Prove that none of the integers $11, 111, 1111, 11111, \ldots$ is a square of an integer. How about $2, 22, 222, 2222, \ldots, 3, 33, 333, 3333, \ldots$; etc.?