Math 307-02: Abstract Algebra, Spring 2017

Instructor: Chi-Kwong Li 李志光

Meeting time and place: TT 2:00 - 3:20 p.m. Jones 306
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Office hours: TWT 11:00 - 12:00 or by appointments.

Course description: The goal of the course is to introduce the basic concepts of (abstract) algebraic structure and techniques.
The course will cover Chapters 1 - 17 and selected material from later chapters such as finite fields, and Cayley graphs, of the following required textbook.

- Joseph A. Gallian, Contemporary Abstract Algebra, Newest edition, Brooks/Cole, Boston, 2013. http://www.amazon.com/Contemporary-Abstract-Algebra-Joseph-Gallian/dp/1133599702

Homework will be assigned every week and due the following Friday (noon).
Homework sessions will be conducted on Thursday, 3:30-4:30 p.m. Jones 113 or Jones 131. Of coruse, help will also be available at the office hours.
You have to use LaTex to typset mathematical document, an excellent skill to acquire. You may get the programs for free.

- For windows,
  (a) download the MikTex program from http://miktex.org/download;
  (b) then download the Texmaker program from http://www.xm1math.net/texmaker/download.html;
  (c) then open the program "texmaker";
  (d) copy (or download and then open) the "homework01.tex" file to the texmaker window;
  (e) select "LaTex" from the "Quick Build" menu;
  (f) click the "=>" arrow on the left of "Quick Buil" to get the pdf output.
- For Mac users,
  (a) download MacTex from http://tug.org/mactex/;
  (b) open the "texshop" program,
  (c) copy (or download and then open) the "homework01.tex" file to the texshop window;
  (d) change "PlainTex" to "LaTex" at "Typeset" menu;
  (e) click "Typeset" icon to get the pdf output.
- You may also use the online editor Write LaTeX .
- Here is a list of TeX commands for mathematics symbols.

Challenging problems will be assigned from time to time;
extra-credits will be given to successful (or partially successful) attempts.

Assessment

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Quizzes (20 min. each) on Feb 2, 16, March 23, April 6, 20.

Exams:  Mid-term   March 2    1:20 hrs  (2:00-3:30 p.m.)
        Final      May   8    3    hrs  (2:00-5:00 p.m.)

Grades (for homework, quizzes, exams, final grade, etc.):

%: 0 - 60 - 65 - 70 - 75 - 80 - 83 - 87 - 90 - 93 - 100
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F	D	C-	C	C+	B-	B	B+	A-	A

Assessment:	Homework	Quizzes	Mid-term	Final
	20%	20%	25%	35%

(Extra credit problems may add another 5%)

Homework list

- Homework 1. Due: Jan 27, noon. [Tex file | pdf file | Sample solution]

Class notes (To be updated)

- Notes on Chapters 0 - 2.
- Notes on Chapters 3 - 4.
- Notes on Chapters 5 - 7.
- Notes on Chapters 8 - 9.
- Notes on Chapters 10 - 11.
- Notes on Chapters 12 - 13.
- Notes on Chapters 14 - 15.
- Notes on Chapters 16 - 17.
- Some techniques in abstract algebra.

Scans of hand written notes in lectures

- Class notes of Jan. 19.
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Additional material

- Sample quizzes and solutions.
- Exam. 1 with solution.
- Sample final examination questios with solutions.
- Sample final examination solutions (both inclass and take home paarts).

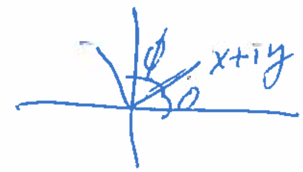
Chapter 0 Preliminaries

Assumption You are familiar with the material in Chapter 0 (Math 214 material).

Notation: $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$.

$\mathbb{C} = \{z = x+iy : x, y \in \mathbb{R}\}$

Known results:



- Well-ordering principle; mathematical induction; complex numbers.
- Division algorithm on $\mathbb{Z}$.
- Greatest common divisor $\gcd(a, b) = xa + yb$ with $x, y \in \mathbb{Z}$.
- A prime p divides ab implies $p|a$ or $p|b$;
- Fundamental Theorem of arithmetic.
- Functions (injective, surjective, bijective, composite function, inverse function, images, preimages).
- Equivalence relations (reflexive, symmetric, and transitive); partitions.
- modular arithmetic on $\mathbb{Z}_n = \{\overline{0}, \ldots, \overline{n-1}\}$ with operations $+$ and $\cdot$ modulo n.

$f : A \to B$

For every $a \in A$, there is a unique $b \in B$ such that $f(a) = b$.

$\mathbb{Z}_n = \{[0], [1], \ldots, [n-1]\}$

$[k] = \{nq + k : q \in \mathbb{Z}\}$; $k = 0, 1, \ldots, n-1$.

$\mathbb{Z}_{12} = \{[0], [1], \ldots, [11]\}$.

$[i] + [j] = [i+j]$, $\quad [2] + [11] = [13] = [1]$

$$[11][11] = [121] = [1]$$
$$\| $$
$$[-1][-1]$$
$$\|$$
$$[1]$$

Checking your readiness

- Please review Chapter 0 and your notes in Math 214.
- You are not ready if you have difficulty in these topics.
- Check your readiness by doing Homework 1.
- If you have troubles in doing it. Come to homework session, form study group, ...

$$3x = 5$$

$$3^{-1}(3x) = 3^{-1} \times 5$$

$$(3^{-1}3)x = 3^{-1} \times 5$$

$$x = 3^{-1}5 = \frac{5}{3}$$

$$\underset{n\times n}{A}\ \underset{n\times n}{X} = \underset{n\times n}{B}$$

$$A^{-1}(AX) = A^{-1}B$$

$$(A^{-1}A)X = A^{-1}B$$

$$X = A^{-1}B$$

Goal of this course

- Learn basic algebraic structures/proof techniques to study advanced mathematics and other subjects.
- What is algebra (vs. analysis, geometry, etc.)?
- Algebra concerns the study of *algebraic structures* arising in number systems, geometrical symmetry, quantum physics!
- An algebraic structure is a set of objects (such as numbers, or symmetric transformations, or function/matrix operations) with one or more (binary) operations.
- Examples

$$\mathbb{N} = \mathbb{Z}^+,\ \mathbb{Z},\ \mathbb{Q},\ \mathbb{Q}^+,\ \mathbb{Q}^*,\ \mathbb{R},\ \mathbb{R}^+,\ \mathbb{R}^*,\ \mathbb{C},\ \mathbb{C}^*,\ M_n(\mathbb{R}).$$

Why Abstract?

$1+2=3$

$a+b=c$

$1+x=3$

$x^2=4$

Example of algebraic structure

Set of bijections on $\{1,2,3\}$

$$S_3 = \left\{ \begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}, \begin{pmatrix}1&2&3\\1&3&2\end{pmatrix}, \begin{pmatrix}1&2&3\\2&1&3\end{pmatrix}, \begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}, \begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}, \begin{pmatrix}1&2&3\\3&2&1\end{pmatrix} \right\}$$

$$\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix} \circ f = \begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}$$

$$\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix} \circ \begin{pmatrix}1&2&3\\2&1&3\end{pmatrix} = \begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}$$

$g \circ f$

$$f \circ f \circ f = \begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}$$

Examples of groups

We will first focus on algebraic structure with only one operation.

- The set of integer under addition.

  (G0) Closed. (G1) Associative. (G2) Identity. (G3) Inverse.

- The set of positive real numbers under multiplication.

- The set $\mathbb{Z}_n = \{\overline{0}, \overline{1}, \ldots, \overline{n-1}\}$ under addition modulo n.

- The set of permutations over $S = \{1, \ldots, n\}$ under function compositions.

  (G4) Commutative (Abelian).

$(G, *)$

(G0) If $x, y \in G$. then $x * y = z \in G$

$(\mathbb{N}, +)$ is closed.

$(\mathbb{N}, -)$ is not closed.

$(\mathbb{Z}, +)$ is closed

$(\mathbb{Z}, -)$ is closed

(G1) If $x, y, z \in G$ s.t. then

$$(x*y)*z = x*(y*z)$$

(G2) Identity:

There is $e \in G$ such that

$e * x = x * e = x$ for every $x \in G$

(G3) Inverse

For every $x \in G$.

there is $y \in G$

so that $x * y = y * x = e$

$3 + 8 + 7 = 18$

$(3 - 8) - 7$

$\neq$

$3 - (8 - 7)$

Motivation

$a x = b$

$\hat{a} * (a * x)$

$= \hat{a} * b$

$(\hat{a} * a) * x$

$= \hat{a} * b$

(G4) If $x, y \in G$ $x * y = y * x$.

Chapter 1 Symmetry of squares and regular polygons

Examples of symmetry group and subgroup

- For a square, there are rotation symmetries: $R_0, R_{90}, R_{180}, R_{270}$, reflection symmstries: H, V, D, D'.
- These operations will "permute" the four corners of the square labeled by 1, 2, 3, 4, and generate 8 different permutations $\begin{pmatrix} 1 & 2 & 3 & 4 \\ i_1 & i_2 & i_3 & i_4 \end{pmatrix}$ in S_4 (the group of all permutations of $\{1, 2, 3, 4\}$. See the table in p. 33.

8 bijection.

- The eight opertaions will form the dihedral group D_4 under composition.
- In general, for an regular n-side polygon with $n \geq 3$, we can form a **dihedral group** D_n.