$\frac{31}{31}$

MATH309: Intermediate Linear Algebra
Homework 1

5 **2.** $z_1 = 0$, $z_2 = x - y$. For $z_1 = 0$, it's obvious that $Az_1 = 0$.

Assume $A = [u_1 | \cdots | u_n]$, $x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$, $y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$. Since $Ax = Ay$, $u_1x_1 + \cdots + u_nx_n = u_1y_1 + \cdots + u_ny_n$. So $u_1(x_1 - y_1) + \cdots + u_n(x_n - y_n) = 0$, which means $A(x - y) = Az_2 = 0$.

5 **6.** Since these are corners of parallelograms, the sum of two corners must be equal to the sum of the other two. So the three possible corners are: $x_1 = (1,1) + (4,2) - (1,3) = (4,0)$, $x_2 = (1,1) - (4,2) + (1,3) = (-2,2)$, $x_3 = -(1,1) + (4,2) + (1,3) = (4,4)$.

5 **9.** Since the column space is all of $\mathbb{R}_3$ (3), m has to be 3. So the rank should be $\leqslant 3$, but there has to be 3 linearly independent columns to span the whole $\mathbb{R}_3$, so $r = 3$, and $n \geqslant 3$.

10. $C_1 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, $C_2 = \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix}$.

11.

$$A_1 = \begin{bmatrix} 1 & 3 & -2 \\ 3 & 9 & -6 \\ 2 & 6 & -4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 3 & -2 \end{bmatrix} = C_1R_1.$$

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$$A_2 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix} = C_2R_2.$$

14. A counterexample would prove the first two.

Let $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$. They have the same column space, but the row space is different. Also, they have different basic columns. However, their ranks must be the same because if they are not, it's impossible for A and B to have the same column space.

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