On the other hand, the point ($x_1 = 15, x_2 = 70$) is not in the feasible region, because even though $x_1 = 15$ and $x_2 = 70$ satisfy (2), (4), (5), and (6), they fail to satisfy (3): $15 + 70$ is not less than or equal to 80. Any point that is not in an LP's feasible region is said to be an **infeasible point.** As another example of an infeasible point, consider ($x_1 = 40$, $x_2 = -20$). Although this point satisfies all the constraints and the sign restriction (5), it is infeasible because it fails to satisfy the sign restriction (6), $x_2 \geq 0$. The feasible region for the Giapetto problem is the set of possible production plans that Giapetto must consider in searching for the optimal production plan.

DEFINITION ■ For a maximization problem, an **optimal solution** to an LP is a point in the feasible region with the largest objective function value. Similarly, for a minimization problem, an optimal solution is a point in the feasible region with the smallest objective function value. ■

Most LPs have only one optimal solution. However, some LPs have no optimal solution, and some LPs have an infinite number of solutions (these situations are discussed in Section 3.3). In Section 3.2, we show that the unique optimal solution to the Giapetto problem is ($x_1 = 20, x_2 = 60$). This solution yields an objective function value of

$$z = 3x_1 + 2x_2 = 3(20) + 2(60) = \$180$$

When we say that ($x_1 = 20, x_2 = 60$) is the optimal solution to the Giapetto problem, we are saying that no point in the feasible region has an objective function value that exceeds 180. Giapetto can maximize profit by building 20 soldiers and 60 trains each week. If Giapetto were to produce 20 soldiers and 60 trains each week, the weekly profit would be \$180 less weekly fixed costs. For example, if Giapetto's only fixed cost were rent of \$100 per week, then weekly profit would be $180 - 100 = \$80$ per week.

PROBLEMS

Group A

1 Farmer Jones must determine how many acres of corn and wheat to plant this year. An acre of wheat yields 25 bushels of wheat and requires 10 hours of labor per week. An acre of corn yields 10 bushels of corn and requires 4 hours of labor per week. All wheat can be sold at \$4 a bushel, and all corn can be sold at \$3 a bushel. Seven acres of land and 40 hours per week of labor are available. Government regulations require that at least 30 bushels of corn be produced during the current year. Let x_1 = number of acres of corn planted, and x_2 = number of acres of wheat planted. Using these decision variables, formulate an LP whose solution will tell Farmer Jones how to maximize the total revenue from wheat and corn.

2 Answer these questions about Problem 1.

- **a** Is ($x_1 = 2, x_2 = 3$) in the feasible region?
- **b** Is ($x_1 = 4, x_2 = 3$) in the feasible region?
- **c** Is ($x_1 = 2, x_2 = -1$) in the feasible region?
- **d** Is ($x_1 = 3, x_2 = 2$) in the feasible region?

3 Using the variables x_1 = number of bushels of corn produced and x_2 = number of bushels of wheat produced, reformulate Farmer Jones's LP.

4 Truckco manufactures two types of trucks: 1 and 2. Each truck must go through the painting shop and assembly shop. If the painting shop were completely devoted to painting Type 1 trucks, then 800 per day could be painted; if the painting shop were completely devoted to painting Type 2 trucks, then 700 per day could be painted. If the assembly shop were completely devoted to assembling truck 1 engines, then 1,500 per day could be assembled; if the assembly shop were completely devoted to assembling truck 2 engines, then 1,200 per day could be assembled. Each Type 1 truck contributes \$300 to profit; each Type 2 truck contributes \$500. Formulate an LP that will maximize Truckco's profit.

Group B

5 Why don't we allow an LP to have $<$ or $>$ constraints?