In the rest of this chapter, we lead the reader through the formulation of several more complicated linear programming models. The most important step in formulating an LP model is the proper choice of decision variables. If the decision variables have been properly chosen, the objective function and constraints should follow without much difficulty. Trouble in determining an LP's objective function and constraints is usually the result of an incorrect choice of decision variables.

PROBLEMS

Group A

Identify which of Cases 1–4 apply to each of the following LPs:

1

$$\begin{aligned} \max z &= x_1 + x_2 \\ \text{s.t.} \quad x_1 + x_2 &\le 4 \\ x_1 - x_2 &\ge 5 \\ x_1, x_2 &\ge 0 \end{aligned}$$

2

$$\begin{aligned} \max z &= 4x_1 + x_2 \\ \text{s.t.} \quad 8x_1 + 2x_2 &\le 16 \\ 5x_1 + 2x_2 &\le 12 \\ x_1, x_2 &\ge 0 \end{aligned}$$

3

$$\begin{aligned} \max z &= -x_1 + 3x_2 \\ \text{s.t.} \quad x_1 - x_2 &\le 4 \\ x_1 + 2x_2 &\ge 4 \\ x_1, x_2 &\ge 0 \end{aligned}$$

4

$$\begin{aligned} \max z &= 3x_1 + x_2 \\ \text{s.t.} \quad 2x_1 + x_2 &\le 6 \\ x_1 + 3x_2 &\le 9 \\ x_1, x_2 &\ge 0 \end{aligned}$$

5 True or false: For an LP to be unbounded, the LP's feasible region must be unbounded.

6 True or false: Every LP with an unbounded feasible region has an unbounded optimal solution.

7 If an LP's feasible region is not unbounded, we say the LP's feasible region is bounded. Suppose an LP has a bounded feasible region. Explain why you can find the optimal solution to the LP (without an isoprofit or isocost line) by simply checking the z-values at each of the feasible region's extreme points. Why might this method fail if the LP's feasible region is unbounded?

8 Graphically find all optimal solutions to the following LP:

$$\begin{aligned} \min z &= x_1 - x_2 \\ \text{s.t.} \quad x_1 + x_2 &\le 6 \\ x_1 - x_2 &\ge 0 \\ x_2 - x_1 &\ge 3 \\ x_1, x_2 &\ge 0 \end{aligned}$$

9 Graphically determine two optimal solutions to the following LP:

$$\begin{aligned} \min z &= 3x_1 + 5x_2 \\ \text{s.t.} \quad 3x_1 + 2x_2 &\ge 36 \\ 3x_1 + 5x_2 &\ge 45 \\ x_1, x_2 &\ge 0 \end{aligned}$$

Group B

10 Money manager Boris Milkem deals with French currency (the franc) and American currency (the dollar). At 12 midnight, he can buy francs by paying .25 dollars per franc and dollars by paying 3 francs per dollar. Let x_1 = number of dollars bought (by paying francs) and x_2 = number of francs bought (by paying dollars). Assume that both types of transactions take place simultaneously, and the only constraint is that at 12:01 A.M. Boris must have a nonnegative number of francs and dollars.

a Formulate an LP that enables Boris to maximize the number of dollars he has after all transactions are completed.

b Graphically solve the LP and comment on the answer.

3.4 A Diet Problem

Many LP formulations (such as Example 2 and the following diet problem) arise from situations in which a decision maker wants to minimize the cost of meeting a set of requirements.