Math 323 **Final Examination** Name: Sample Solution

1. Two plants supply three customers with medical supplies. The unit costs of shipping from the plants to the customers, along with the supplies and demands, are given in the table on the right.

From	To: Customer 1	Customer 2	Customer 3	Supply
Plant 1	\$55	\$65	\$80	35
Plant 2	\$10	\$15	\$25	50
Demand	10	10	10	

(a) The companys goal is to minimize the cost of meeting customers demands.
Find TWO optimal bfs for this transportation problem.

(b) What is the range of shipping cost from plant 2 to customer 1 so that the current optimal solution remains unchanged.

(c) If customer 2s demand increased by one unit, how much would the cost increase?

a)

	$v_1=10$	$v_2=15$	$v_3=25$	$v_4=0$	
$u_1=0$	[55] -45	[65] -50	[80] -55	[0] 35	35
$u_2=0$	[10] 10	[15] 10	[25] 10	[0] 50	50
	10	10	10	55	

← Initial solution by minimum cost method

~~Optimal cost: 100+150+250=500~~

Checking $u_i+v_j-c_{ij}$ with $u_1=0$ we see that the solution is optimal since $u_i+v_j-c_{ij}<0$ for all non-basic variables, there is a $\boxed{\text{unique optimal bfs.}}$ So finding another one is impossible.

b) If we change c_{21} from 10 to $10+\Delta$ then we change v_1 to $10+\Delta$ in the above table. and only $u_1+v_1-c_{11}$ will change and become $10+\Delta-55$. To keep it ≤ 0, we need $\boxed{\Delta \le 45.}$ So as long as $\boxed{\Delta \le 45, \text{ i.e., } c_{21} \le 55}$ the current solution remains optimal.

c) If we change the demand as said, we get

	$v_1=10$	$v_2=15$	$v_3=25$	$v_4=0$	
$u_1=0$	[55]	[65]	[80]	[0] 35	35
$u_2=0$	[10] 10	[15] 11	[25] 10	[0] 19	50
	10	11	10	54	

The cost will change by $\boxed{15}$

2. Telephone calls from New York to Los Angeles are transported as follows: The call is sent first to either Chicago or Memphis, then routed through either Denver or Dallas, and finally sent to Los Angeles. The number of phone lines joining each pair of cities is shown in the table on the right.

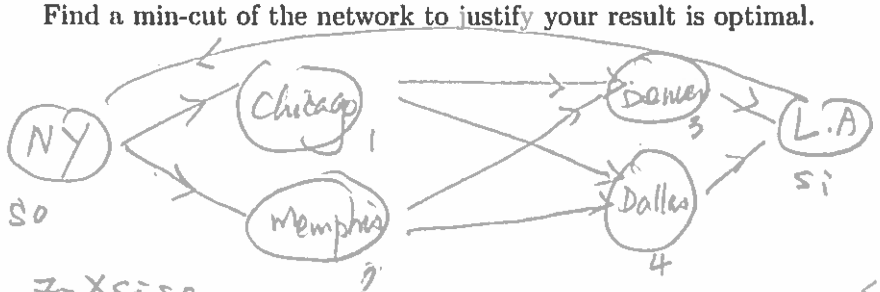
Cities	No. of Telephone Lines
N.Y.–Chicago	500
N.Y.–Memphis	400
Chicago–Denver	300
Chicago–Dallas	250
Memphis–Denver	200
Memphis–Dallas	150
Denver–L.A.	400
Dallas–L.A.	350

(a) Formulate an LP that can be used to determine the maximum number of calls that can be sent from New York to Los Angeles at any given time.

(b) Use the Ford-Fulkerson method to determine the maximum number of calls that can be sent from New York to Los Angeles at any given time.

Find a min-cut of the network to justify your result is optimal.

a)



Max $Z = x_{si,so}$

s.t. $x_{so,1} \le 500$, $x_{1,3} \le 400$, $x_{1,4} \le 250$, $x_{2,3} \le 200$, $x_{2,4} \le 150$

$x_{3,si} \le 400$, $x_{4,si} \le 350$ all variables ≥ 0

$x_{so,1} + x_{so,2} - x_{si,so} = 0$

$x_{14} + x_{13} - x_{so,1} = 0$

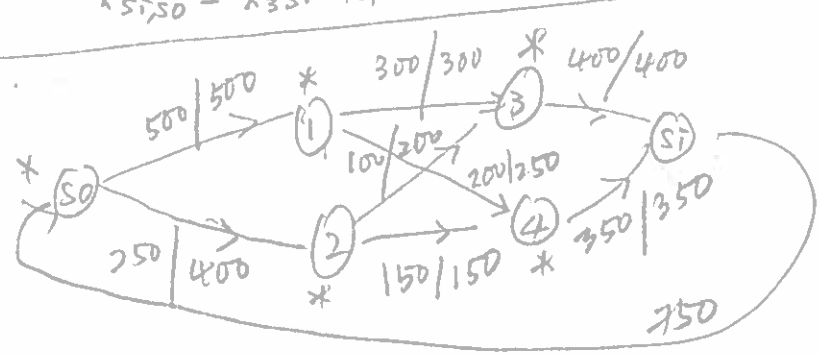
$x_{23} + x_{24} - x_{so,2} = 0$

$x_{3,si} - x_{13} - x_{2,3} = 0$

$x_{4,si} - x_{14} - x_{2,4} = 0$

$x_{si,so} - x_{3si} - x_{4si} = 0$

Consider the flow:

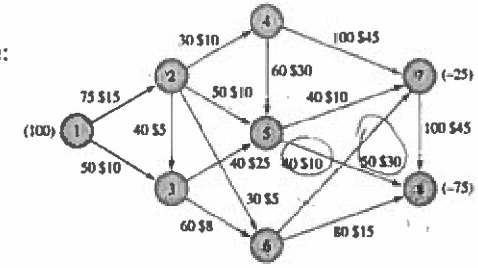


The flow equals 750

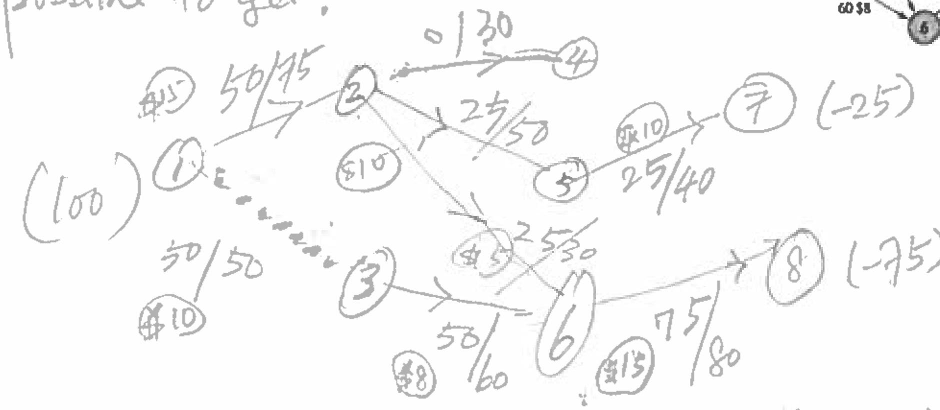
$S = \{so, 1, 2, 3, 4\}$, $\bar{S} = \{si\}$ $K(S, \bar{S}) = 750$

So we have the max flow.

3. Find the optimal solution to the MCNFP in the figure:



Use the minimum cost edges if possible to get:



Including the ··→· edge and the artificial edge (1,4) we get a basic feasible solution

set $\boxed{y_1=0}$, $y_1-y_2=15$, $\boxed{y_2=-15}$; $y_2-y_4=10$, $\boxed{y_4=-25}$

$y_2-y_5=10$, $\boxed{y_5=-25}$, $y_5-y_7=10$, $\boxed{y_7=-35}$

$y_2-y_6=5$, $\boxed{y_6=-20}$, $y_3-y_6=8$, $\boxed{y_3=-12}$, $y_6-y_8=15$, $\boxed{y_8=-35}$

Then

$\hat{c}_{13} = y_1-y_3-c_{13} = 0+12-10=2>0$ ✓ (upper bound variable)

$\hat{c}_{23} = y_2-y_3-c_{23} = -15+12-5=-8<0$ ✓ (lower bound variable)

$\hat{c}_{35} = y_3-y_5-c_{35} = -12+25-25=-12<0$ ✓

$\hat{c}_{45} = y_4-y_5-c_{45} = -15+25-30=-20<0$ ✓

$\hat{c}_{47} = y_4-y_7-c_{47} = -15+35-45=-25<0$ ✓

$\hat{c}_{58} = y_5-y_8-c_{58} = -25+35-10=0$ ✓

$\hat{c}_{67} = y_6-y_7-c_{67} = -20+35-30=-15$ ✓

$\hat{c}_{78} = y_7-y_8-c_{78} = -35+35-45=-45$ ✓

So we get the optimal solution

$50\times10 + 50\times8 + 75\times15$

$+\ 50\times15 + 25\times10 + 25\times5 + 25\times10 = 3400$

4. Use the implicit enumeration method to find the optimal solution to the following 01 IP:

$$\max z = 5x_1 - 7x_2 + 10x_3 + 3x_4 - x_5$$

subject to $\quad -x_1 - 3x_2 + 3x_3 - x_4 - 2x_5 \le 0 \qquad (1)$

subject to $\quad 2x_1 - 5x_2 + 3x_3 - 2x_4 - 2x_5 \le 3 \qquad (2)$

subject to $\quad -x_2 + x_3 + x_4 - x_5 \ge 2 \qquad (3)$

All variables 0 or 1.

Check optimal: $(x_1, x_2, x_3, x_4, x_5) = (1, 0, 1, 1, 0)$.

But this violates (1).

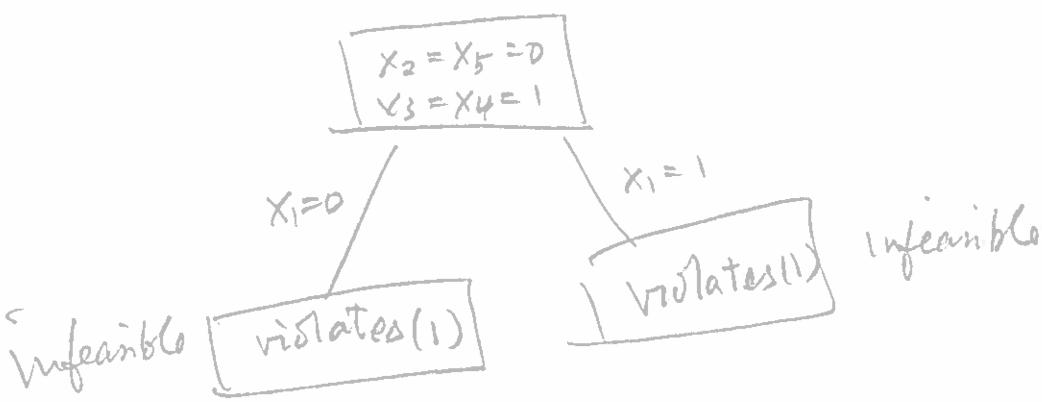
Check feasibility.

$(1, 1, 0, 1, 1)$ satisfies (1)

$(0, 1, 0, 1, 1)$ satisfies (2)

$(1, 0, 1, 1, 0)$ satisfies (3)

But note that (3) holds means $(x_2, x_3, x_4, x_5) = (0, 1, 1, 0)$.



So the problem is infeasible

5. A soda delivery truck starts at location 1 and must deliver soda to locations 2, 3, 4, and 5 before returning to location 1. The distance between these locations is given in the table. The soda truck wants to minimize the total distance traveled.

	Location				
Location	1	2	3	4	5
1	0	20	4	10	25
2	20	0	5	30	10
3	4	5	0	6	6
4	10	25	6	0	20
5	35	10	6	20	0

(a) Replace each diagonal entry of the distance matrix by a big M, and solve the assignment problem by the Hungarian method. Show that the solution does not yield a tour.

(b) If we also replace the $(5,2)$ entry of the distance matrix by M, would the solution of the assignment problem yield a tour?

(c) Use the branch and bound method to find the optimal tour.

(a)

$$\begin{pmatrix} M & 20 & 4 & 10 & 25 \\ 20 & M & 5 & 30 & 10 \\ 4 & 5 & M & 6 & 6 \\ 10 & 25 & 6 & M & 20 \\ 35 & 10 & 6 & 20 & M \end{pmatrix} \to \begin{pmatrix} M & 16 & 0 & 6 & 21 \\ 15 & M & 0 & 25 & 5 \\ 0 & 1 & M & 2 & 2 \\ 4 & 19 & 0 & M & 14 \\ 29 & 4 & 0 & 14 & M \end{pmatrix} \to \begin{pmatrix} M & 12 & 0 & 2 & 17 \\ 11 & M & 0 & 21 & 1 \\ 0 & 1 & M & 2 & 2 \\ 0 & 15 & 0 & M & 10 \\ 25 & 0 & 0 & 10 & M \end{pmatrix}$$

$$\to \begin{pmatrix} M & 11 & 0 & 1 & 16 \\ 11 & M & 0 & 20 & 0 \\ 0 & 0 & M & 1 & 1 \\ 0 & 14 & 0 & M & 9 \\ 26 & 0 & 1 & 10 & M \end{pmatrix} \to \begin{pmatrix} M & 11 & 0 & \boxed{0} & 16 \\ 11 & M & 0 & 19 & \boxed{0} \\ \boxed{0} & 0 & M & 0 & 1 \\ 0 & 14 & \boxed{0} & M & 9 \\ 26 & \boxed{0} & 1 & 9 & M \end{pmatrix}$$

We get the solution
1 − 4 − 3 − 1
2 − 5 − 2.

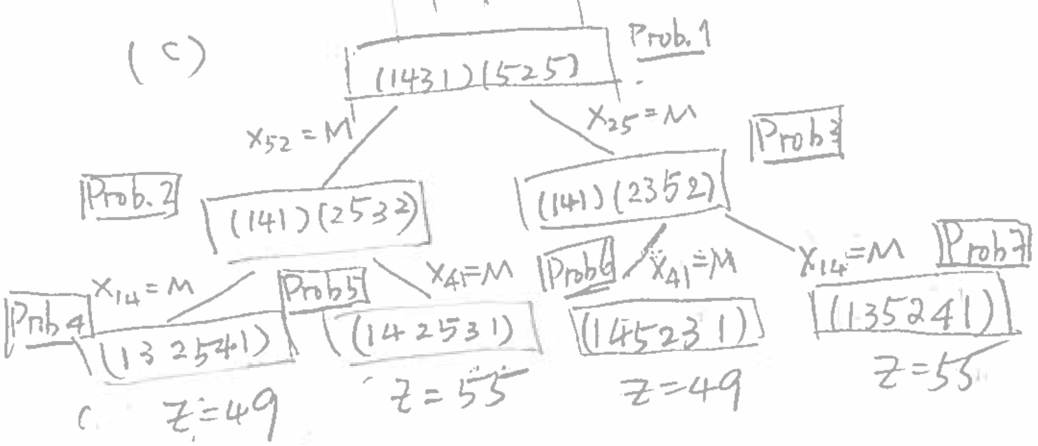
(b)

$$\begin{pmatrix} M & 20 & 4 & 10 & 25 \\ 20 & M & 5 & 30 & 10 \\ 4 & 5 & M & 6 & 6 \\ 10 & 25 & 6 & M & 20 \\ 35 & M & 6 & 20 & M \end{pmatrix} \to \begin{pmatrix} M & 16 & 0 & 6 & 21 \\ 15 & M & 0 & 25 & 5 \\ 0 & 1 & M & 2 & 2 \\ 4 & 19 & 0 & M & 14 \\ 29 & M & 0 & 14 & M \end{pmatrix} \to \begin{pmatrix} M & 12 & 0 & 2 & 17 \\ 14 & M & 0 & 21 & 1 \\ 0 & 1 & M & 2 & 2 \\ 0 & 15 & 0 & M & 10 \\ 25 & M & 0 & 10 & M \end{pmatrix}$$

$$\to \begin{pmatrix} M & 11 & 0 & 1 & 16 \\ 14 & M & 0 & 20 & 0 \\ 0 & 0 & M & 1 & 1 \\ 0 & 14 & 0 & M & 9 \\ 25 & M & 0 & 9 & M \end{pmatrix} \to \begin{pmatrix} M & 11 & 0 & 0 & 16 \\ 14 & M & 0 & 19 & 0 \\ 0 & 0 & M & 0 & 1 \\ 0 & 14 & 0 & M & 9 \\ 25 & M & 0 & 8 & M \end{pmatrix}$$

We get the solution
1 − 4 − 1
2 − 5 − 3 − 2.

(c)



Thus optimal solution is

(132541) or (145231)

Z = 49

Prob 4

M	20	4	M	25
20	M	5	30	10
4	5	M	6	6
10	25	6	M	20
35	M	6	20	M

→

M	16	0	M	21
15	M	0	25	5
0	1	M	2	2
4	19	0	M	14
29	M	0	14	M

→

M	15	0	M	20
15	M	0	24	4
0	0	M	1	1
4	18	0	M	13
29	M	0	13	M

→

M	11	0	M	16
11	M	0	20	0
0	0	M	1	1
0	14	0	M	9
25	M	9	9	M

→

M	11	0	M	15
12	M	1	20	0
0	0	M	0	0
0	14	0	M	8
25	M	0	8	M

→

M	3	0	M	7
12	M	9	20	0
0	0	M	0	0
0	14	8	M	8
17	M	0	0	M

tour: 1-3-2-5-4-1

$z = 4+5+10+20+10 = 49$ //

Prob 5

M	20	4	10	25
20	M	5	30	10
4	5	M	6	6
M	25	6	M	20
35	M	6	20	M

→

M	16	0	6	21
15	M	0	25	5
0	1	M	2	2
M	19	0	M	14
29	M	0	14	M

→

M	15	0	4	19
15	M	0	23	3
0	0	M	0	0
M	18	0	M	12
29	M	0	12	M

→

M	12	0	1	16
12	M	0	20	0
0	0	M	0	0
M	15	0	M	9
26	M	0	9	M

→

M	11	0	0	15
12	M	1	20	0
0	0	M	0	0
M	14	0	M	8
25	M	0	8	M

→

M	0	0	0	15
1	M	1	20	0
0	0	M	0	0
M	3	0	M	8
14	M	0	8	M

→

M	0	1	0	16
0	M	1	19	0
0	0	M	0	1
M	2	0	M	8
14	M	0	7	M

tour: 1-4-2-5-3-1

$z = 10+25+10+6+4 = 55$ //

Prob 6 (This is the transpose of Problem 4)

M	20	4	10	25
20	M	5	30	M
4	5	M	6	6
M	25	6	M	20
35	10	6	20	M

→

M	16	0	6	21
15	M	0	25	M
0	1	M	2	2
M	19	0	M	14
29	4	0	14	M

→

M	15	0	4	19
15	M	0	23	M
0	0	M	0	0
M	18	9	M	12
29	3	9	12	M

→

M	12	0	1	16
12	M	0	20	M
0	0	M	0	0
M	15	0	M	9
26	0	0	9	M

→

M	11	0	0	15
11	M	0	19	M
0	0	M	0	0
M	14	0	M	8
26	0	1	9	M

→

M	11	0	0	7
3	M	0	19	M
0	8	M	8	0
M	14	0	M	0
18	0	1	9	M

tour: 1-4-5-2-3-1

$Z = 10+20+10+5+4$
$= 49$ //

Prob 7

M	20	4	M	25
20	M	5	30	M
4	5	M	6	6
10	25	6	M	20
35	10	6	20	M

(This is the transpose of Problem 5)

→ tour 1-3-5-2-4-1

$Z = 4+6+10+25+10 = 55$ //