Math 323 Homework 3

4.5 3).

Standard Form:

max $Z = 2\ x_1 -\ x_2 +\ x_3$ subject to $3\ x_1 +\ x_2 +\ x_3 + s_1 = 60$, $x_1 -\ x_2 + 2\ x_3 + s_2 = 50$, $x_1 +\ x_2 -\ x_3 + s_3 = 20$ and $x_1,\ x_2,\ x_3 \geq 0$.

Initial Tableau:

C_B	B	$(+2)\ x_1$	$(-1)\ x_2$	$(+1)\ x_3$	$(+0)\ s_1$	$(+0)\ s_2$	$(+0)\ s_3$	constraints
0	s_1	3	1	1	1	0	0	60
0	s_2	1	-1	2	0	1	0	10
0	s_3	1	1	-1	0	0	1	20
	C	2	-1	1	0	0	0	Z=0

Final Tableau:

C_B	B	$(+2)\ x_1$	$(-1)\ x_2$	$(+1)\ x_3$	$(+0)\ s_1$	$(+0)\ s_2$	$(+0)\ s_3$	constraints
0	s_1	0	0	1	1	-1	-2	10
2	x_1	1	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	15
-1	x_2	0	1	$-\frac{3}{2}$	0	$-\frac{1}{2}$	$\frac{1}{2}$	5
	C	0	0	$-\frac{3}{2}$	0	$-\frac{3}{2}$	$-\frac{1}{2}$	Z=25

So the optimal solution is $x_1 = 15$, $x_2 = 5$, z=25.

4.6 3).

Standard Form:

max $Z_1 = -2\ x_1 + 5\ x_2$ subject to $3\ x_1 + 8\ x_2 + s_1 = 12$, $2\ x_1 + 3\ x_2 + s_2 = 6$, and $x_1,\ x_2 \geq 0$.

Initial Tableau:

C_B	B	$(-2)\ x_1$	$(+5)\ x_2$	$(+0)\ s_1$	$(+0)\ s_2$	constraints
0	s_1	3	8	1	0	12
0	s_2	2	3	0	1	6
	C	-2	5	0	0	$Z_1 = 0$

Final Tableau:

C_B	B	$(-2)\ x_1$	$(+5)\ x_2$	$(+0)\ s_1$	$(+0)\ s_2$	constraints
5	x_2	$\frac{3}{8}$	1	$\frac{1}{8}$	0	$\frac{3}{2}$
0	s_2	$\frac{7}{8}$	0	$-\frac{3}{8}$	1	$\frac{3}{2}$
	C	$-\frac{31}{8}$	0	$-\frac{5}{8}$	0	$Z_1 = \frac{15}{2}$

So the optimal solution is $x_1 = 0$, $x_2 = \frac{3}{2}$, $Z_1 = \frac{15}{2}$. Therefore $Z = -\frac{15}{2}$.

4.11 1). Standard Form:

max $Z = 5\ x_1 + 3\ x_2$ subject to $4\ x_1 + 2\ x_2 + s_1 = 12$, $4\ x_1 + x_2 + s_2 = 10$, $x_1 + x_2 + s_3 = 4$ and $x_1,\ x_2, s_1, s_2, s_3 \geq 0$.

Initial Tableau:

C_B	B	$(+5)\ x_1$	$(+3)\ x_2$	$(+0)\ s_1$	$(+0)\ s_2$	$(+0)\ s_3$	constraints
0	s_1	4	2	1	0	0	12
0	s_2	4	1	0	1	0	10
0	s_3	1	1	0	0	1	4
	C	5	3	0	0	0	$Z_1 = 0$

Final Tableau:

C_B	B	$(+5)\ x_1$	$(+3)\ x_2$	$(+0)\ s_1$	$(+0)\ s_2$	$(+0)\ s_3$	constraints
3	x_2	0	1	1	0	0	2
5	x_1	1	0	$-\frac{1}{4}$	$\frac{1}{4}$	0	2
0	s_3	0	0	-1	$-\frac{1}{3}$	$\frac{4}{3}$	0
	C	0	0	$\frac{1}{4}$	$-\frac{5}{4}$	0	$Z = 16$

From the last row of the final tableau, the basic variable $s_3 = 0$, so this tableau is degenerate.

4.12 3). Standard Form:

max $Z = 3\,x_1 + x_2 + 0\,e_1 + 0\,s_1 - M(a_1 + a_2)$ subject to $x_1 + x_2 - e_1 + a_1 = 3$, $2\,x_1 + x_2 + s_1 = 4$, $x_1 + x_2 + a_2 = 3$ and $x_1,\ x_2, e_1, a_1, s_1, a_2 \geq 0$.

Initial Tableau:

C_B	B	$(+3)\ x_1$	$(+1)\ x_2$	$(+0)\ s_1$	$(+0)\ s_2$	$(-M)\ s_3$	$(-M) s_4$	constraints
-M	a_1	1	1	-1	0	1	0	3
0	s_1	2	1	0	1	0	0	4
-M	a_2	1	1	0	0	0	1	3
	C	3+2M	1+2M	-M	0	0	0	$Z = -6M$

Final Tableau:

C_B	B	$(+3)\ x_1$	$(+1)\ x_2$	$(+0)\ s_1$	$(+0)\ s_2$	$(-M)\ s_3$	$(-M) s_4$	constraints
1	x_2	0	1	-2	-1	1	0	2
3	x_1	1	0	1	0	-1	0	1
-M	a_2	0	0	1	0	-1	1	0
	C	0	0	M-1	1	-1	0	$Z = 5$

The optimal solution is $x_1 = 1,\ \ x_2 = 2,\ Z = 5$.

4.14 2) Standard Form:

max $Z = 2\ x_1 + (x_2' - x_2'')$ subject to $3\ x_1 + x_2' - x_2'' + s_1 = 6$, $x_1 + x_2' - x_2'' + s_2 = 4$, and $x_1,\ x_2', x_2'', s_1, s_2 \geq 0$.

Initial Tableau:

C_B	B	$(+2)\ x_1$	$(+1)\ x_2'$	$(-1)\ x_2''$	$(+0)\ s_1$	$(+0)\ s_2$	constraints
0	s_1	3	1	-1	1	0	6
0	s_2	1	1	-1	0	1	4
	C	2	1	-1	0	0	$Z = 0$

Final Tableau:

C_B	B	$(+2)\ x_1$	$(+1)\ x_2'$	$(-1)\ x_2''$	$(+0)\ s_1$	$(+0)\ s_2$	constraints
2	x_1	1	0	0	0.5	-0.5	1
1	x_2'	0	1	-1	-0.5	1.5	3
	C	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$Z = 5$

The optimal solution is $x_1 = 1,\ \ x_2 = 3,\ Z = 5$.