Math 323 Homework 4

The Diet Problem

Standard Form:

min $Z = 50\,x_1 + 20\,x_2 + 30\,x_3 + 80\,x_4$ subject to $400\,x_1 + 200\,x_2 + 150\,x_3 + 500\,x_4 - e_1 = 500$, $3\,x_1 + 2\,x_2 - e_2 = 6$, $2\,x_1 + 2\,x_2 + 4\,x_3 + 4\,x_4 - e_3 = 10$, $2\,x_1 + 4\,x_2 + 1\,x_3 + 5\,x_4 - e_4 = 8$ and $x_1,\ x_2,\ x_3,\ x_4, e_1, e_2, e_3, e_4 \geq 0$.

Initial Tableau:

C_B	B	(50) x_1	(20) x_2	(30) x_3	(80) x_4	(+0) e_1	(+0) e_2	(+0) e_3	(+0)e_4	Constraints
0	e_1	400	200	150	500	-1	0	0	0	500
0	e_2	3	2	0	0	0	-1	0	0	6
0	e_3	2	2	4	4	0	0	-1	0	10
0	e_4	2	4	1	5	0	0	0	-1	8
	C	50	20	30	80	0	0	0	0	Z=0

Final Tableau:

C_B	B	(50) x_1	(20) x_2	(30) x_3	(80) x_4	(+0) e_1	(+0) e_2	(+0) e_3	(+0)e_4	Constraints
0	e_1	-137.5	0	0	-350	1	-62.5	-37.5	0	250
20	x_2	1.5	1	0	0	0	-0.5	0	0	3
30	x_3	-0.25	0	1	1	0	0.25	-0.25	0	1
0	e_4	3.75	0	0	-4	0	-1.75	-0.25	1	5
	C	27.5	0	0	50	0	2.5	7.5	0	Z=90

So the optimal solution is $x_2 = 3,\ \ x_3 = 1,\ z = 90.$ ($x_1,\ x_4 = 0$)

Questions

1. Yes. After changing the coefficients in front of x_1 and x_4 to 70 and 60 respectively, their corresponding numbers in last row changed to 47.5 and 30, so the current basis is still optimal.

2. No. After changing the coefficients in front of x_1 and x_4 to 20 and 100, respectively, their corresponding numbers in last row changed to -2.5 and 70, so the current basis is no longer optimal.

3. Yes. After b_1 is changed to 800 and b_4 is reduced to 3, the new tableau gives an optimal basis of x_2 and x_3.

4. Yes. After b_4 is changed to 6 and b_1 is changed to 600, I went through the simplex algorithm again and obtained the same basis.

5. Yes. After changing c_4 to 60 and changing c_3 to 15, last row is now [23.75, 0, 0, 45, 0, 6.25, 3.75, 0], so the current basis remains optimal. The new optimal solution is $x_2 = 3,\ \ x_1 = 1$, and $z = 75$.

6. After changing b_1 to 60 and b_2 to 6, the new final tableau is

C_B	B	(50) x_1	(20) x_2	(30) x_3	(80) x_4	(+0) e_1	(+0) e_2	(+0) e_3	(+0)e_4	Constraint
0	e_1	-137.5	0	0	-350	1	-62.5	-37.5	0	815
20	x_2	1.5	1	0	0	0	-0.5	0	0	4
30	x_3	-0.25	0	1	1	0	0.25	-0.25	0	0.5
0	e_4	3.75	0	0	-4	0	-1.75	-0.25	1	8.5
	C	23.75	0	0	50	0	2.5	7.5	0	Z=95

So the optimal basis is still x_2 and x_3.