11 Consider the following LP:

$$\max z = 10x_1 + x_2$$
$$\text{s.t.} \quad x_1 \leq 1$$
$$20x_1 + x_2 \leq 100$$
$$x_1, x_2 \geq 0$$

a Find all the basic feasible solutions for this LP.

b Show that when the simplex is used to solve this LP, every basic feasible solution must be examined before the optimal solution is found.

By generalizing this example, Klee and Minty (1972) constructed (for $n = 2, 3, \ldots$) an LP with n decision variables and n constraints for which the simplex algorithm examines $2^n - 1$ basic feasible solutions before the optimal solution is found. Thus, there exists an LP with 10 variables and 10 constraints for which the simplex requires $2^{10} - 1 = 1{,}023$ pivots to find the optimal solution. Fortunately, such "pathological" LPs rarely occur in practical applications.

12 Productco produces three products. Each product requires labor, lumber, and paint. The resource requirements, unit price, and variable cost (exclusive of raw materials) for each product are given in Table 70. Currently, 900 labor hours, 1,550 gallons of paint, and 1,600 board feet of lumber are available. Additional labor can be purchased at $6 per hour, additional paint at $2 per gallon, and additional lumber at $3 per board foot. For the following two sets of priorities, use preemptive goal programming to determine an optimal production schedule. For set 1:

Priority 1 Obtain profit of at least $10,500.
Priority 2 Purchase no additional labor.
Priority 3 Purchase no additional paint.
Priority 4 Purchase no additional lumber.

For set 2:

Priority 1 Purchase no additional labor.
Priority 2 Obtain profit of at least $10,500.
Priority 3 Purchase no additional paint.
Priority 4 Purchase no additional lumber.

13 Jobs at Indiana University are rated on three factors:

Factor 1 Complexity of duties
Factor 2 Education required
Factor 3 Mental and or visual demands

For each job at IU, the requirement for each factor has been rated on a scale of 1–4, with a 4 in factor 1 representing high complexity of duty, a 4 in factor 2 representing high educational requirement, and a 4 in factor 3 representing high mental and/or visual demands.

TABLE 70

Product	Labor	Lumber	Paint	Price ($)	Variable Cost ($)
1	1.5	2	3	26	10
2	3	3	2	28	6
3	2	4	2	31	7

REVIEW PROBLEMS

All problems from Sections 5.2 and 5.3 are relevant, along with Chapter 5 Review Problems 1, 2, 6, and 7.

Group A

1 Consider the following LP and its optimal tableau (Table 51):

$$\max z = 4x_1 + x_2$$
$$\text{s.t.} \quad x_1 + 2x_2 = 6$$
$$x_1 - x_2 \geq 3$$
$$2x_1 + x_2 \leq 10$$
$$x_1, x_2 \geq 0$$

a Find the dual of this LP and its optimal solution.

b Find the range of values of b_3 for which the current basis remains optimal. If $b_3 = 11$, what would be the new optimal solution?

2 For the LP in Problem 1, graphically determine the range of values on c_1 for which the current basis remains optimal. (*Hint:* The feasible region is a line segment.)

3 Consider the following LP and its optimal tableau (Table 52):

$$\max z = 5x_1 + x_2 + 2x_3$$
$$\text{s.t.} \quad x_1 + x_2 + x_3 \leq 6$$
$$6x_1 + x_3 \leq 8$$
$$x_2 + x_3 \leq 2$$
$$x_1, x_2, x_3 \geq 0$$

a Find the dual to this LP and its optimal solution.

b Find the range of values of c_1 for which the current basis remains optimal.

c Find the range of values of c_2 for which the current basis remains optimal.

4 Carco manufactures cars and trucks. Each car contributes $300 to profit and each truck, $400. The resources required to manufacture a car and a truck are shown in Table 53. Each day, Carco can rent up to 98 Type 1 machines at a cost of $50 per machine. The company now has 73 Type 2 machines and 260 tons of steel available. Marketing considerations dictate that at least 88 cars and at least 26 trucks be produced. Let

$X1$ = number of cars produced daily
$X2$ = number of trucks produced daily
$M1$ = type 1 machines rented daily

To maximize profit, Carco should solve the LP given in Figure 11. Use the LINDO output to answer the following questions:

a If cars contributed $310 to profit, what would be the new optimal solution to the problem?

b What is the most that Carco should be willing to pay to rent an additional Type 1 machine for 1 day?

c What is the most that Carco should be willing to pay for an extra ton of steel?

d If Carco were required to produce at least 86 cars, what would Carco's profit become?

e Carco is considering producing jeeps. A jeep contributes $600 to profit and requires 1.2 days on machine 1, 2 days on machine 2, and 4 tons of steel. Should Carco produce any jeeps?

5 The following LP has the optimal tableau shown in Table 54.

$$\max z = 4x_1 + x_2$$
$$\text{s.t.} \quad 3x_1 + x_2 \geq 6$$
$$2x_1 + x_2 \geq 4$$
$$x_1 + x_2 = 3$$
$$x_1, x_2 \geq 0$$

a Find the dual of this LP and its optimal solution.

b Find the range of values of the objective function coefficient of x_2 for which the current basis remains optimal.

c Find the range of values of the objective function coefficient of x_1 for which the current basis remains optimal.

6 Consider the following LP and its optimal tableau (Table 55):

$$\max z = 3x_1 + x_2 - x_3$$
$$\text{s.t.} \quad 2x_1 + x_2 + x_3 \leq 8$$
$$4x_1 + x_2 - x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0$$

a Find the dual of this LP and its optimal solution.

b Find the range of values of b_2 for which the current basis remains optimal. If $b_2 = 12$, what is the new optimal solution?

TABLE 51

z	x_1	x_2	e_2	s_3	a_1	a_2	rhs
1	0	0	0	$\frac{7}{3}$	$M - \frac{2}{3}$	M	$\frac{58}{3}$
0	0	1	0	$-\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{2}{3}$
0	1	0	0	$\frac{2}{3}$	$-\frac{1}{3}$	0	$\frac{14}{3}$
0	0	0	1	1	-1	-1	1

TABLE 52

z	x_1	x_2	x_3	s_1	s_2	s_3	rhs
1	0	$\frac{1}{6}$	0	0	$\frac{5}{6}$	$\frac{7}{6}$	9
0	0	$\frac{1}{6}$	0	1	$-\frac{1}{6}$	$-\frac{5}{6}$	3
0	1	$-\frac{1}{6}$	0	0	$\frac{1}{6}$	$-\frac{1}{6}$	1
0	0	1	1	0	0	1	2

TABLE 53

Vehicle	Days on Type 1 Machine	Days on Type 2 Machine	Tons of Steel
Car	0.8	0.6	2
Truck	1	0.7	3

TABLE 55

z	x_1	x_2	x_3	s_1	s_2	rhs
1	0	0	1	$\frac{1}{2}$	$\frac{1}{2}$	9
0	0	1	3	2	-1	6
0	1	0	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1