Homework 5

Chapter 4 Review A #1
Set 1:
MATLAB Output:

Optimal solution found.
x = 150.0000 0 337.5000 0 0 475.0000 50.0000 0 50.0000 0 0
fval = 50.0000

Therefore, $Z = 50$
$x_1 = 150, x_2 = 0, x_3 = 337.5, s_1^{-1} = 0, s_1^+ = 0, s_2^{-1} = 475, s_2^+ = 50, s_3^{-1} = 0, s_4^- = 0, s_4^+ = 0.$

Set 2:
MATLAB Output:

Optimal solution found.
x = 150.0000 0 337.5000 0 0 475.0000 50.0000 0 50.0000 0 0
fval = 50.0000

Therefore, $Z = 50$
$x_1 = 150, x_2 = 0, x_3 = 337.5, s_1^{-1} = 0, s_1^+ = 0, s_2^{-1} = 475, s_2^+ = 50, s_3^{-1} = 0, s_4^- = 0, s_4^+ = 0.$

Chapter Six Review

1.
a) $minw = 6y_1 + 3y_2 10y_3$
Subject to: $y_1 + y_2 + 2y_3 \geq 4$
$2y_1 - y_2 + y_3 \geq 1$
$y_1, y_2 \leq 0, y_3 \geq 0$

Optimal Solution:
$y_1 = -2/3$, $y_2 = 0$, $y_3 = 7/3$
$w = 58/3$

b) When $b_3 = 9$, $2x_1 + x_2 \leq 9$ goes through the point $(4, 1)$. Were $b_3 < 9$ then the current basis would not be optimal. Similarly, if $b_3 = 12$ then we reach the point $(6, 0)$ and any b_3 greater than that would no longer remain optimal either. Thus the current basis remains optimal when $9 \leq b_3 \leq 12$.
If $b_3 = 11$, then $x_1 = 16/3, x_2 = 1/3$, and $z = 65/3$

2.
Since the slope of our first constraint $(x_1 + 2x + 2 = 6)$ is $1/2$, we deduce that the current basis remains optimal for $c_1 \geq 1/2$. The feasible solution is the line segment from A to B on the graph attached.

3.a) $min w = 6y_1 + 8y_2 + 2y_3$
Subject to $y_1 + 6y_2 \geq 5$
$y_1 + y_3 \geq 1$
$y_1 + y_2 + y_3 \geq 2$
$y_1, y_2, y_3 \geq 0$

Optimal Solution:
$y_1 = 0, y_2 = 5/6, y_3 = 7/6, w = 9$

b)

$$B^{-1} = \begin{bmatrix} 1 & -1/6 & -5/6 \\ 0 & 1/6 & -1/6 \\ 0 & 0 & 1 \end{bmatrix}$$

$c_1 = 5 + \Delta, c_3 = 2, c_4 = 0$

$$cB^{-1} = \begin{bmatrix} 0 & 5+\Delta & 2 \end{bmatrix} \begin{bmatrix} 1 & -1/6 & -5/6 \\ 0 & 1/6 & -1/6 \\ 0 & 0 & 1 \end{bmatrix}$$

$1/6 - 1/6\Delta \geq 0$ when $\Delta \leq 1$
$5/6 + 1/6\Delta \geq 0$ when $\Delta \geq -5$
$7/6 - 1/6\Delta \geq 0$ when $\Delta \leq 7$
Thus $-5 \leq \Delta \leq 1$ and current basis remains optimal for $0 \leq c_1 \leq 6$

c)
$c_2 = 1 + \Delta$

$$\begin{bmatrix} 0 & 5/6 & 7/6 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - [1 + \Delta]$$

$\Delta \leq 1/6$
Therefore, current basis remains optimal when $c_2 \leq 7/6$

6.
a) $min w = 8y_1 + 10y_2$
Subject to: $2y_1 + 4y_2 \geq 3$
$y_1 + y_2 \geq 1$
$y_1 - y_2 \geq -1$
$y_1, y_2 \geq 0$

Optimal Solution:
$y_1 = 1/2, y_2 = 1/2, w = 9$

b)

$$B^{-1}b = \begin{bmatrix} 2 & -1 \\ -1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 8 \\ 10 + \Delta \end{bmatrix} = \begin{bmatrix} 6-\Delta \\ 1 + \Delta/2 \end{bmatrix}$$

We use this to determine that $8 \leq b_2 \leq 16$
If $b_2 = 12$, then the optimal solution is $z = 14, x_1 = 4, x_2 = 2$

Chapter 6 Review
Problem 2