TABLE 57

z	x_1	x_2	s_1	s_2	rhs
1	0	0	0	1	10
0	0	$\frac{1}{3}$	1	$-\frac{2}{3}$	$\frac{4}{3}$
0	1	$\frac{7}{3}$	0	$\frac{1}{3}$	$\frac{10}{3}$

18 Consider the following LP and its optimal tableau (Table 57):

(10 points)

$$\max z = 3x_1 + 2x_2$$
$$\text{s.t.} \quad 2x_1 + 5x_2 \le 8$$
$$3x_1 + 7x_2 \le 10$$
$$x_1, x_2 \ge 0$$

a Find the dual of this LP and its optimal solution.

b Find the range of values of b_2 for which the current basis remains optimal. Also find the new optimal solution if $b_2 = 5$.

TABLE 58

z	x_1	x_2	x_3	s_1	s_2	rhs
1	8	1	0	0	2	16
0	2	2	0	1	−1	4
0	6	1	1	0	1	8

TABLE 59

z	x_1	x_2	s_1	a_2	s_2	s_3	rhs
1	0	0	0	1	$M-1$	$M+\frac{3}{2}$	$\frac{9}{2}$
0	0	0	1	0	0	$-\frac{1}{2}$	$\frac{1}{2}$
0	0	1	0	−2	2	$-\frac{3}{2}$	$\frac{3}{2}$
0	1	0	0	1	−1	1	1

TABLE 60

z	x_1	x_2	a_1	s_2	s_1	s_3	rhs
1	0	0	0	$\frac{1}{5}$	M	$M-\frac{7}{5}$	$-\frac{18}{5}$
0	0	1	0	$\frac{3}{5}$	0	$-\frac{1}{5}$	$\frac{6}{5}$
0	1	0	0	$-\frac{1}{5}$	0	$\frac{2}{5}$	$\frac{3}{5}$
0	0	0	1	1	−1	1	0

(20 points)

13 Consider the following LP and its optimal tableau (Table 58):

(10 points)

$$\max z = -4x_1 + x_2 + 2x_3$$
$$\text{s.t.} \quad 8x_1 + 3x_2 + x_3 \le 2$$
$$6x_1 + x_2 + x_3 \le 8$$
$$x_1, x_2, x_3 \ge 0$$

a Find the dual to this LP and its optimal solution.

b Find the range of values of the objective function coefficient of x_3 for which the current basis remains optimal.

c Find the range of values of the objective function coefficient of x_1 for which the current basis remains optimal.

14 Consider the following LP and its optimal tableau (Table 59):

(10 points)

$$\max z = 3x_1 + x_2$$
$$\text{s.t.} \quad 2x_1 + x_2 \le 4$$
$$3x_1 + 2x_2 \ge 6$$
$$4x_1 + 2x_2 = 7$$
$$x_1 \ge 0, x_2 \ge 0$$

a Find the dual to this LP and its optimal solution.

b Find the range of values of the right-hand side of the third constraint for which the current basis remains optimal. Also find the new optimal solution if the right-hand side of the third constraint were $\frac{15}{2}$.

15 Consider the following LP:

(10 points)

$$\max z = 3x_1 + x_2$$
$$\text{s.t.} \quad 4x_1 + x_2 \le 7$$
$$5x_1 + 2x_2 \le 10$$
$$x_1, x_2 \ge 0$$

The optimal solution to this LP is $z = \frac{17}{3}, x_1 = \frac{4}{3}, x_2 = \frac{5}{3}$. Use the graphical approach to determine the range of values for the right-hand side of the second constraint for which the current basis remains optimal.

19 Consider the following LP:

(5 points)

$$\max z = -2x_1 + 6x_2$$
$$\text{s.t.} \quad x_1 + x_2 \ge 2$$
$$-x_1 + x_2 \le 1$$
$$x_1, x_2 \ge 0$$

This LP is unbounded. Use this fact to show that the following LP has no feasible solution:

$$\min 2y_1 + y_2$$
$$\text{s.t.} \quad y_1 - y_2 \ge -2$$
$$y_1 + y_2 \ge 6$$
$$y_1 \le 0, y_2 \ge 0$$

22 Radioco manufactures two types of radios. The only scarce resource that is needed to produce radios is labor. The company now has two laborers. Laborer 1 is willing to work up to 40 hours per week and is paid $5 per hour. Laborer 2 is willing to work up to 50 hours per week and is paid $6 per hour. The price as well as the resources required to build each type of radio are given in Table 61.

a Letting x_i be the number of type i radios produced each week, show that Radioco should solve the following LP (its optimal tableau is given in Table 62):

TABLE 61

Radio 1		Radio 2	
Price ($)	Resource Required	Price ($)	Resource Required
25	Laborer 1: 1 hour Laborer 2: 2 hours Raw material cost: $5	22	Laborer 1: 2 hours Laborer 2: 1 hours Raw material cost: $4

TABLE 62

z	x_1	x_2	s_1	s_2	rhs
1	0	0	$\frac{1}{3}$	$\frac{4}{3}$	80
0	1	0	$-\frac{1}{3}$	$\frac{2}{3}$	20
0	0	1	$\frac{2}{3}$	$-\frac{1}{3}$	10

$$\max z = 3x_1 + 2x_2$$
$$\text{s.t.} \quad x_1 + 2x_2 \le 40$$
$$2x_1 + x_2 \le 50$$
$$x_1, x_2 \ge 0$$

b For what values of the price of a Type 1 radio would the current basis remain optimal?

c For what values of the price of a Type 2 radio would the current basis remain optimal?

d If laborer 1 were willing to work only 30 hours per week, would the current basis remain optimal?

e If laborer 2 were willing to work as many as 60 hours per week, would the current basis remain optimal?

f If laborer 1 were willing to work an additional hour, what is the most that Radioco should pay?

g If laborer 2 were willing to work only 48 hours, what would Radioco's profits be? Verify your answer by determining the number of radios of each type that would be produced.

h A Type 3 radio is under consideration for production. The specifications of a Type 3 radio are as follows: price, $30; 2 hours from laborer 1; 2 hours from laborer 2; cost of raw materials, $3. Should Radioco manufacture any Type 3 radios?