Sample Solution

Math 323 Operations Research

10a. min $W = 8y_1 + 10y_2$
Subject to:
$2y_1 + 3y_2 \geq 3$
$5y_1 + 7y_2 \geq 2$
$y_1, y_2 \geq 0$

Optimal solution of the dual system is $(y_1, y_2) = C_B^T * B^{-1} = (0, 1)$, of the primal system is $(x_1, x_2) = (10/3, 0)$ and $W = Z = 10$

10b. We can get $B^{-1} = \begin{pmatrix} 1 & -2/3 \\ 0 & 1/3 \end{pmatrix}$ and $b = \begin{pmatrix} 8 \\ 10+\Delta \end{pmatrix}$. $B^{-1}b = \begin{pmatrix} (4-2\Delta)/3 \\ (10+\Delta)/3 \end{pmatrix}$. With $(4 - 2\Delta)/3, (10+\Delta)/3 \geq 0$, we can get $-10 \leq \Delta \leq 2$. Thus the current basis remains optimal when $0 \leq b_2 \leq 12$.

When $b_2 = 5, B^{-1}b = \begin{pmatrix} 14/3 \\ 5/3 \end{pmatrix}$. Thus the optimal solution is $(x_1, x_2) = (5/3, 0)$ with $z = 5$

13a. min $W = 2y_1 + 8y_2$
Subject to :
$8y_1 + 6y_2 \geq 4$
$3y_1 + y_2 \geq 1$
$y_1 + y_2 \geq 2$
$y_1, y_2 \geq 0$
Optimal solution of the dual system is $(y_1, y_2) = C_B^T * B^{-1} = (0, 2)$, of the primal system is $(x_1, x_2, x_3) = (0, 0, 8)$ and $W = Z = 16$

13b. C_B is now $\begin{pmatrix} 0 \\ 2+\Delta \end{pmatrix}$, $C = c^T - C_B^T B^{-1} A = (4, 1, 0) - (0, 2+\Delta) * \begin{pmatrix} 2 & 2 & -1 \\ 6 & 1 & 1 \end{pmatrix} = (-8 - 6\Delta, -1 - \Delta, -2 - \Delta)$. With $-8 - 6\Delta, -1 - \Delta, -2 - \Delta \leq 0$, we can get $\Delta \geq -1$. Thus when the current basis remains optimal, $c_3 \geq 1$.

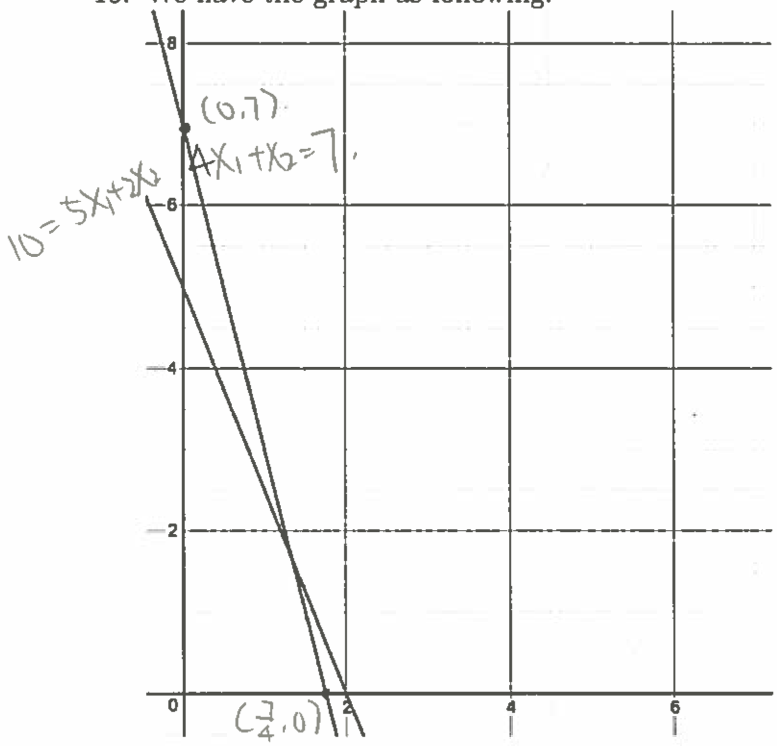
13c. C_B is still $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$. $C_1 = c - C_B^T B^{-1} A_1 = 4 + \Delta - 12 = -8 + \Delta$. With $-8 + \Delta \leq 0$, we get $\Delta \leq 8$. Thus when the current basis remains optimal, $c_1 \leq 12$.

14a. min $W = 4y_1 + 6y_2 + 7y_3$
Subject to :
$2y_1 + 3y_2 + 4y_3 \geq 3$
$y_1 + 2y_2 + 2y_3 \geq 1$
$y_1 \geq 0, y_2 \leq 0, y_3$ is urs.
Optimal solution of the dual system is $(y_1, y_2, y_3) = C_B^T * B^{-1} = (0, -1, 3/2)$, of the primal system is $(x_1, x_2, x_3) = (1, 3/2)$ and $W = Z = 9/2$

14b. We can get $B^{-1} = \begin{pmatrix} 1 & 0 & -1/2 \\ 0 & 2 & -3/2 \\ 0 & -1 & 1 \end{pmatrix}$ and $b = \begin{pmatrix} 4 \\ 6 \\ 7+\Delta \end{pmatrix}$. $B^{-1}b = \begin{pmatrix} (1-\Delta)/2 \\ (3-3\Delta)/2 \\ 1+\Delta \end{pmatrix}$. With $(1-\Delta)/2, (3-3\Delta)/2, 1+\Delta \geq 0$, we can get $-1 \leq \Delta \leq 1$. Thus the current basis remains optimal when $6 \le b_2 \le 8$.

When $b_2 = 15/2$, $B^{-1}b = \begin{pmatrix} 1/4 \\ 3/4 \\ 3/2 \end{pmatrix}$. Thus the optimal solution is $(x_1, x_2) = (3/2, 3/4)$ with $z = 21/4$.

15. We have the graph as following.



The current basis remains optimal, as long as the graph of $5x_1 + 2x_2 = 10$ has no intersection with $4x_1 + x_2 = 7$ on y-axis. Thus we have $b_2 \le 14$. no intercepts on x-axis, Thus $\frac{35}{4} \le b_2$.

$\therefore \frac{35}{4} \le b_2 \le 14$

19. The dual of the above Lp is:
min $W = 2y_1 + y_2$
Subject to:
$y_1 - y_2 \ge -2$
$y_1 + y_2 \ge 6$
$y_1 \le 0, y_2 \ge 0,$
The dual LP is the same as the following LP below. From the theory, since the primal LP is unbounded, the dual LP is infeasible. Thus the following LP has no feasible solution.

22a. x_1 is the number of radio 1, and x_2 is the number of radio 2.
We want to max the profit, which is the Z = price of the radio - labor cost - raw material cost = $(25 - 1*5 + 2*6 - 5)x_1 + (22 - 2*5 - 1*6 - 4)x_2 = 3x_1 + 2x_2$
We want have to limit the labor hour. For labor 1: $x_1 + 2x_2 \le 40$; for labor 2: $2x_1 + x_2 \le 50$.
Thus we have the following LP:
max $Z = 3x_1 + 2x_2$
Subject to:
$x_1 + 2x_2 \le 40$

$2x_1 + x_2 \le 50$
$x_1, x_2 \ge 0$

22b. Let C_B to be $\begin{pmatrix} 3+\Delta \\ 2 \end{pmatrix}$, $C = c^T - C_B^T B^{-1} A = (0,0) - (3+\Delta, 2) * \begin{pmatrix} -1/3 & 2/3 \\ 2/3 & -1/3 \end{pmatrix} =$ $((\Delta - 1)/3, (-2\Delta - 4)/3)$. With $(\Delta - 1)/3, (-2\Delta - 4)/3 \le 0$, we can get $-2 \le \Delta \le 1$. Thus when the current basis remains optimal, $23 \le$ Type1 price ≤ 26, $1 \le c_1 \le 4$.

22c. Let C_B to be $\begin{pmatrix} 3 \\ 2+\Delta \end{pmatrix}$, $C = c^T - C_B^T B^{-1} A = (0,0) - (3, 2+\Delta) * \begin{pmatrix} -1/3 & 2/3 \\ 2/3 & -1/3 \end{pmatrix} =$ $((-1 - 2\Delta)/3, (\Delta - 4)/3)$. With $(-1 - 2\Delta)/3, (\Delta - 4)/3 \le 0$, we can get $-1/2 \le \Delta \le 4$. Thus when the current basis remains optimal, $21.5 \le$ Type2 price ≤ 26, $3/2 \le c_1 \le 6$.

22d. We can get the new $B^{-1}b = \begin{pmatrix} -1/3 & 2/3 \\ 2/3 & -1/3 \end{pmatrix} * \begin{pmatrix} b_1 \\ 50 \end{pmatrix} = \begin{pmatrix} (-b_1 + 100)/3 \\ (2b_1 - 50)/3 \end{pmatrix}$. With $(-b_1 + 100)/3, (2b_1 - 50)/3 \ge 0$, we can get $25 \le b_1 \le 100$, and that is when the current basis remains optimal.
Since now $b_1 = 30$, and it is in the range. Thus the current basis remains optimal.

22e. We can get the new $B^{-1}b = \begin{pmatrix} -1/3 & 2/3 \\ 2/3 & -1/3 \end{pmatrix} * \begin{pmatrix} 40 \\ b_2 \end{pmatrix} = \begin{pmatrix} (-40 + 2b_2)/3 \\ (80 - b_2)/3 \end{pmatrix}$. With $(-40 + 2b_2)/3, (80 - b_2)/3 \ge 0$, we can get $20 \le b_2 \le 80$, and that is when the current basis remains optimal.
Since now $b_1 = 60$, and it is in the range. Thus the current basis remains optimal.

22f. The shadow price is $C_B^T B^{-1} = (3,2) * \begin{pmatrix} -1/3 & 2/3 \\ 2/3 & -1/3 \end{pmatrix} = (1/3, 4/3)$
Thus if laborer 1 were willing to work an additional hour, the most that Radioco should pay is \$1/3.

22g. According to 22e, now $B^{-1}b = \begin{pmatrix} 56/3 \\ 32/3 \end{pmatrix}$.
Thus the number of radios of each type that would be produced is $(x_1, x_2) = (56/3, 32/3)$ and the new profit is $Z = 3x_1 + 2x_2 = 77.33$. To make it be integer, $(x_1, x_2) = (19, 10)$. $Z = 57 + 20 = 77$

22h. The new LP is now:
max $Z = 3x_1 + 2x_2 + 5x_3$
Subject to:
$x_1 + 2x_2 + 2x_3 \le 40$
$2x_1 + x_2 + 2x_3 \le 50$
$x_1, x_2, x_3 \ge 0$
We have $A_3 = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ and thus $C_3 = 5 - (3,2) * \begin{pmatrix} -1/3 & 2/3 \\ 2/3 & -1/3 \end{pmatrix} * \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 5 - 10/3 = 5/3$. Since C_3 is positive, x_3 should be a basic variable to maximize the profit. Thus Radioco should manufacture Type 3 radios.