

1.

	C1	C2	C3	Supply
W1	\$15	\$35	\$25	$\leq 40$
W2	\$10	\$50	\$40	$\leq 30$
Dummy	\$90	\$80	\$220	$\leq 20$
Demand	$\geq 30$	$\geq 30$	$\geq 30$	

NW Rule:

	C1	C2	C3	Supply
W1	15 30	35 10	25	40
W2	10	50 20	40 10	30
Dummy	90	80	110 20	20
Demand	30	30	30	Cost=4,400

Minimum Cost Rule:

	C1	C2	C3	Supply
W1	15	35 10	25 30	40
W2	10 30	50 0	40	30
Dummy	90	80 20	110	20
Demand	30	30	30	Cost=3,000

Vogel's Method:

	C1	C2	C3	Supply
W1	15	35 10	25 30	40
W2	10 30	50 0	40	30
Dummy	90	80 20	110	20
Demand	30	30	30	Cost=3,000

Check Optimal:

	C1	C2	C3	Supply
W1	15	35	25	40
W2	10	50	40	30
Dummy	90	80	110	20
Demand	30	30	30	Cost=3,000

$$\begin{aligned}
 u_1 &= 0 & v_1 &= -5 \\
 u_2 &= 15 & v_2 &= 35 \\
 u_3 &= 45 & v_3 &= 25
 \end{aligned}$$

Yes it is optimal, and the minimum cost is 3000.

2.

	C1	C2	C3	Supply
W1	\$15	\$35	\$25	$\leq 40$
W2	\$10	\$50	\$40	$\leq 30$
Dummy	\$100	\$100	\$100	$\leq 20$
Demand	$\geq 30$	$\geq 30$	$\geq 30$	

NW Rule:

	C1	C2	C3	Supply
W1	15	35	25	40
W2	10	50	40	30
Dummy	100	100	100	20
Demand	30	30	30	Cost=4,200

Minimum Cost Rule:

	C1	C2	C3	Supply
W1	15	35	25	40
W2	10	50	40	30
Dummy	100	100	100	20
Demand	30	30	30	Cost=3,400

Vogel's Method:

	C1	C2	C3	Supply
W1	15	35	25	40
		10	30	
W2	10	50	40	30
	30	0		
Dummy	100	100	100	20
		20		
Demand	30	30	30	Cost=3,400

Check Optimal:

	C1	C2	C3	Supply
W1	15	35	25	40
		10	30	
W2	10	50	40	30
	30	0		
Dummy	100	100	100	20
		20		
Demand	30	30	30	Cost=3,400

$$u_1 = 0$$

$$v_1 = -5$$

$$u_2 = 15$$

$$v_2 = 35$$

$$u_3 = 65$$

$$v_3 = 25$$

Yes it is optimal, and the minimum cost is 3400.

3.

NW Rule:

	M1	M2	M3	M4	M5	M6	Dummy	Supply
M1RT	7 200	8 0	9	10	11	12	0	200
M1OT	11	12 100	13	14	15	16	0	100
M2RT	M	7 160	8 40	9	10	11	0	200
M2OT	M	11	12 100	13	14	15	0	100
M3RT	M	M	7 100	8 100	9	10	0	200
M3OT	M	M	11	12 100	13	14	0	100
M4RT	M	M	M	7 140	8 60	9	0	200
M4OT	M	M	M	11	12 100	13	0	100
M5RT	M	M	M	M	7 30	8 150	0 20	200
M5OT	M	M	M	M	11	12	0 100	100
M6RT	M	M	M	M	M	7	0 200	200
M6OT	M	M	M	M	M	11	0 100	100
Demand	200	260	240	340	190	150	420	

Minimum Cost Rule:

	M1	M2	M3	M4	M5	M6	Dummy	Supply
M1RT	7 200	8 0	9	10	11	12	0	200
M1OT	11	12 100	13	14	15	16	0	100
M2RT	M	7	8	9	10	11	0 200	200
M2OT	M	11 100	12	13	14	15	0	100
M3RT	M	M	7 180	8	9	10	0 20	200
M3OT	M	M	11 60	12 40	13	14	0	100
M4RT	M	M	M	7	8	9	0 200	200
M4OT	M	M	M	11 100	12	13	0	100
M5RT	M	M	M	M 10	7 190	8	0	200
M5OT	M	M	M	M 100	11	12	0	100
M6RT	M	M 50	M	M	M	7 150	0	200
M6OT	M	M 10	M	M 90	M	11	0	100
Demand	200	260	240	340	190	150	420	

Vogel's Method:

	M1	M2	M3	M4	M5	M6	Dummy	Supply
M1RT	7 200	8 0	9	10	11	12	0	200
M1OT	11	12	13 20	14 80	15	16	0	100
M2RT	M 200	7	8	9	10	11	0	200
M2OT	M	11 60	12 20	13	14	15	0 20	100
M3RT	M	M	7 200	8	9	10	0	200
M3OT	M	M	11	12	13	14	0 <del>100</del>	<del>100</del>
M4RT	M	M	M	7 200	8	9	0	200
M4OT	M	M	M	11	12	13	0 100	100
M5RT	M	M	M	M 10	7 190	8	0	200
M5OT	M	M	M	M	11	12	0 100	100
M6RT	M	M	M	M 50	M	7 150	0	200
M6OT	M	M	M	M	M	11	0 100	100
Demand	200	260	240	340	190	150	420	

Check Optimal:

	M1	M2	M3	M4	M5	M6	Dummy	Supply
M1RT	7 200	8	9	10	11	12	0	200
M1OT	11	12 100	13	14	15	16	0	100
M2RT	M	7 0	8	9	10	11	0 200	200
M2OT	M	11 100	12	13	14	15	0	100
M3RT	M	M	7 180	8	9	10	0 20	200
M3OT	M	M	11 60	12 40	13	14	0	100
M4RT	M	M	M	7	8	9	0 200	200
M4OT	M	M	M	11 100	12	13	0	100
M5RT	M	M	M	M	7 10	8	0 190	200
M5OT	M	M	M	M	11 100	12	0	100
M6RT	M	M 50	M	M	M	7 150	0	200
M6OT	M	M 10	M	M	M	11	0 $\Delta$	100
Demand	200	260	240	340	190	150	420	

Cost=9580+260M. Loop:  $x_{32} \rightarrow x_{37} \rightarrow x_{127} \rightarrow x_{122}$ .  $\Delta=10$ . Updated as below:



	M1	M2	M3	M4	M5	M6	Dummy	Supply
M1RT	7 200	8	9	10	11	12	0	200
M1OT	11	12 100	13	14	15	16	0	100
M2RT	M	7 10	8	9	10	11	0 190	200
M2OT	M	11 100	12	13	14	15	0	100
M3RT	M	M	7 180	8	9	10	0 20	200
M3OT	M	M	11 60	12 40	13	14	0	100
M4RT	M	M	M	7	8	9	0 200	200
M4OT	M	M	M	11 100	12	13	0	100
M5RT	M	M	M	M 10	7 190	8	0	200
M5OT	M	M	M	M 100	11	12	0	100
M6RT	M	M 50	M	M	M	7 150	0	200
M6OT	M	M	M	M 90	M	11	0 10	100
Demand	200	260	240	340	190	150	420	

Cost=9650+250M, which is better. Iterating to get the optimal solution:



	M1	M2	M3	M4	M5	M6	Dummy	Supply	
M1RT	200	7	8	9	10	11	12	0	200
M1OT		11	12	13	14	15	16	0	100
M2RT		M	7	8	9	10	11	0	200
M2OT		M	11	12	13	14	15	0	100
M3RT		M	M	7	8	9	10	0	200
M3OT		M	M	11	12	13	14	0	100
M4RT		M	M	M	7	8	9	0	200
M4OT		M	M	M	11	12	13	0	100
M5RT		M	M	M	M	7	8	0	200
M5OT		M	M	M	M	11	12	0	100
M6RT		M	M	M	M	M	7	0	200
M6OT		M	M	M	M	M	11	0	100
Demand	200	260	240	340	190	150	420		

Now cost=10660, and the solution is optimal.



Good!