

Section 7.5

1 Five employees are available to perform four jobs. The time it takes each person to perform each job is given in Table 50. Determine the assignment of employees to jobs that minimizes the total time required to perform the four jobs.

TABLE 50

Person	Time (hours)			
	Job 1	Job 2	Job 3	Job 4
1	22	18	30	18
2	18	—	27	22
3	26	20	28	28
4	16	22	—	14
5	21	—	25	28

Note: Dashes indicate person cannot do that particular job.

6 Five male characters (Billie, John, Fish, Glen, and Larry) and five female characters (Ally, Georgia, Jane, Rene, and Nell) from *Ally McBeal* are marooned on a desert island. The problem is to determine what percentage of time each woman on the island should spend with each man. For example, Ally could spend 100% of her time with John or she could "play the field" by spending 20% of her time with each man. Table 55 shows a "happiness index" for each potential pairing of a man and woman. For example, if Larry and Rene spend all their time together, they earn 8 units of happiness for the island.

- a Play matchmaker and determine an allocation of each man and woman's time that earns the maximum total happiness for the island. Assume that happiness earned by a couple is proportional to the amount of time they spend together.
- b Explain why the optimal solution to this problem will, for any matrix of "happiness indices," always involve each woman spending all her time with one man.

TABLE 55

	Ally	Georgia	Jane	Rene	Nell
Billie	8	6	4	7	5
John	5	7	6	4	9
Fish	10	6	5	2	10
Glen	1	0	0	0	0
Larry	5	7	9	8	6

Section 7.6

1 General Ford produces cars at L.A. and Detroit and has a warehouse in Atlanta; the company supplies cars to customers in Houston and Tampa. The cost of shipping a car between points is given in Table 60 ("—" means that a shipment is not allowed). L.A. can produce as many as 1,100 cars, and Detroit can produce as many as 2,900 cars. Houston must receive 2,400 cars, and Tampa must receive 1,500 cars.

- a Formulate a balanced transportation problem that can be used to minimize the shipping costs incurred in meeting demands at Houston and Tampa.
- b Modify the answer to part (a) if shipments between L.A. and Detroit are not allowed.
- c Modify the answer to part (a) if shipments between Houston and Tampa are allowed at a cost of \$5.

TABLE 60

From	To (\$)				
	L.A.	Detroit	Atlanta	Houston	Tampa
L.A.	0	140	100	90	225
Detroit	145	0	111	110	119
Atlanta	105	115	0	113	78
Houston	89	109	121	0	—
Tampa	210	117	82	—	0

Chapter 7

10 Find the optimal solution to the balanced transportation problem in Table 72 (minimization).

11 In Problem 10, suppose we increase s_1 to 16 and d_3 to 11. The problem is still balanced, and because 31 units (instead of 30 units) must be shipped, one would think that the total shipping costs would be increased. Show that the total shipping cost has actually decreased by \$2, however. This is called the "more for less" paradox. Explain why increasing both the supply and the demand has decreased cost. Using the theory of shadow prices, explain how one could have predicted that increasing s_1 and d_3 by 1 would decrease total cost by \$2.

TABLE 72

	4	2	4	15
	12	8	4	15
	10	10	10	

18 During the next three quarters, Airco faces the following demands for air conditioner compressors: quarter 1—200; quarter 2—300; quarter 3—100. As many as 240 air compressors can be produced during each quarter. Production costs/compressor during each quarter are given in Table 78. The cost of holding an air compressor in inventory is \$100/quarter. Demand may be backlogged (as long as it is met by the end of quarter 3) at a cost of \$60/compressor/quarter. Formulate the tableau for a balanced transportation problem whose solution tells Airco how to minimize the total cost of meeting the demands for quarters 1–3.

TABLE 78

Quarter 1	Quarter 2	Quarter 3
\$200	\$180	\$240

19 A company is considering hiring people for four types of jobs. It would like to hire the number of people in Table 79 for each type of job.

Four types of people can be hired by the company. Each type is qualified to perform two types of jobs according to

TABLE 79

	Job			
	1	2	3	4
Number of people	30	30	40	20

TABLE 80

Jobs qualified for	Type of Person			
	1	2	3	4
1 and 3	2 and 3	3 and 4	1 and 4	

Table 80. A total of 20 Type 1, 30 Type 2, 40 Type 3, and 20 Type 4 people have applied for jobs. Formulate a balanced transportation problem whose solution will tell the company how to maximize the number of employees assigned to suitable jobs. (Note: Each person can be assigned to at most one job.)