Math 323 Operations Research HW7

Sample Solution

7.5-1:

22	18	30	18	M
18	-	27	22	M
26	20	28	28	M
16	22	-	14	M
21	-	25	28	M

→

4	0	8	0	10
0	-	5	4	10
6	0	4	8	0
2	8	-	0	14
0	-	0	7	7

Thus: Person 1 is assigned Job 2; Person 2 is assigned Job 1; Person 3 has no job; Person 4 is assigned Job 4; and Person 5 is assigned Job 3. The minimum cost is 18+18+14+25=75.

7.5-6(a):

-8	-6	-4	-7	-5
-5	-7	-6	-4	-9
-10	-6	-5	-2	-10
-1	0	0	0	0
-5	-7	-9	-8	-6

→

0	1	3	0	3
4	1	2	4	0
0	3	4	7	0
0	0	0	0	1
5	2	0	1	4

Thus, Billie is going to spend all his time with Rene; John is going to spend all his time with Nell; Fish is going to spend all his time with Ally; Glen is going to spend all his time with Georgia; Larry is going to spend all his time with Jane. The maximum happiness is 7+9+10+0+9=35.

7.5-6(b):

Using transportation problem set up, with the NW rule for the first basic feasible solution:

	Ally	Georgia	Jane	Rene	Nell	Supply
Billie	8: 1	6: 0	4	7	5	1
John	5	7: 1	6: 0	4	9	1
Fish	10	6	5: 1	2: 0	10	1
Glen	1	0	0	0: 1	0: 0	1
Larry	5	7	9	8	6: 1	1
Demand	1	1	1	1	1	

We can see that all the solutions are 1 and 0, since each person can supply 1 and demand 1. Through the iteration process, to maximize the overall happiness, we want to use the people with higher happiness index as much as possible, in this case, Δ can at most be 1. Thus, in the iteration, each Δ will be 1, causing the substituted variable to decrease to 0. Therefore, the final optimal solution will contain all 1 and 0, meaning each person ends up spending their entire time with one partner.

7.6-1(a):

	LA	Detroit	Atlanta	Houston	Tampa	Dummy	Supply
LA	0	140	100	90	225	0	5100
Detroit	145	0	111	110	119	0	6900
Atlanta	105	115	0	113	78	0	4000
Houston	89	109	121	0	M	0	4000
Tampa	210	117	82	M	0	0	4000
Demand	4000	4000	4000	6400	5500	100	

7.6-1(b):

	LA	Detroit	Atlanta	Houston	Tampa	Dummy	Supply
LA	0	M	100	90	225	0	5100
Detroit	M	0	111	110	119	0	6900
Atlanta	105	115	0	113	78	0	4000
Houston	89	109	121	0	M	0	4000
Tampa	210	117	82	M	0	0	4000
Demand	4000	4000	4000	6400	5500	100	

7.6-1(c):

	LA	Detroit	Atlanta	Houston	Tampa	Dummy	Supply
LA	0	140	100	90	225	0	5100
Detroit	145	0	111	110	119	0	6900
Atlanta	105	115	0	113	78	0	4000
Houston	89	109	121	0	5	0	4000
Tampa	210	117	82	5	0	0	4000
Demand	4000	4000	4000	6400	5500	100	

7-10:

4	2	4	
10	5		15
12	8	4	
	5	10	15
10	10	10	C=130

7-11:

4	2	4	
10	6		16
12	8	4	
	4	11	15
10	10	11	C=128

Increasing s_1 and d_3 is the same as changing b for the LP simplex method. Thus, the basic variables remain the same and the only change is the value of the basic variable. Increasing 1 unit of s_1 while d_2 remains the same means that we are able to increase x_{12} by 1, and correspondingly decrease y_{22} by 1. Looking at the cost coefficient, substituting y_{22} by x_{12} for 1 unit will save 6 dollars. Increasing d_3 will cause x_{23} to increase by 1, which cost 4 dollars more. Summing up the two change, 6-4=2, and thus we will save 2 dollars for the overall cost by supplying and demanding 1 more unit.

7-18:

	Qd1	Qd2	Qd3	Dummy	Supply
Qs1	200	300	400	0	240
Qs2	240	180	280	0	240
Qs3	360	300	240	0	240
Demand	200	300	100	120	

After iterating, the optimal solution is:

	Qd1	Qd2	Qd3	Dummy	Supply
Qs1	200 200	300 0	400	0	240
Qs2	240	180 240	280	0	240
Qs3	360	300 60	240 100	0 80	240
Demand	200	300	100	120	C=125200

7-19:

	J1	J2	J3	J4	Supply
P1	0	1	0	1	20
P2	1	0	0	1	30
P3	1	1	0	0	40
P4	0	1	1	0	20
Dummy	1	1	1	1	10
Demand	30	30	40	20	

After iterating, the optimal solution is:

	J1	J2	J3	J4	Supply
P1	0 20	1	0	1	20
P2	1	0 30	0	1	30
P3	1	1	0 40	0	40
P4	0	1	1	0 20	20
Dummy	1 10	1	1	1	10
Demand	30	30	40	20	

The maximum number of people assigned jobs is 110 people.