Math 323 Operations Research HW10

Sample Solution

8.5-1:

Min Z=$4x_{12} + 3x_{24} + 2x_{46} + 3x_{13} + 3x_{35} + 2x_{25} + 2x_{56}$, subject to:

$x_{12} + x_{13} = 1$

$x_{12} = x_{24} + x_{25}$

$x_{13} = x_{35}$

$x_{24} = x_{46}$

$x_{25} + x_{35} = x_{56}$

$x_{46} + x_{56} = 1$

If $x_{ij} = 1$, the shortest path from node 1 to node 6 contains arc (i, j); if $x_{ij} = 0$, tgen shortest path from node 1 to node 6 does not contain arc (i, j).

The solution to this problem is unbounded, meaning there are multiple solutions:

Solution1: $x_{12} = x_{25} = x_{56} = 1$

Solution2: $x_{13} = x_{35} = x_{56} = 1$

The minimized total cost is 8.

8.5-3:

Set Detroit to be node 1, Dallas to be node2, and City 1-3 be node 3-5 respectively. Create node 6 to be a super sink to extract all the excess inventory. The problem is formulated as following:

Min Z =$2800x_{13} + 2600x_{14} + 2300x_{15} + 2300x_{23} + 2000x_{24} + 2000x_{25} + 0x_{16} + 0x_{26}$. Subject to:

$x_{13} \le 2200$; $x_{14} \le 2200$; $x_{15} \le 2200$; $x_{23} \le 2200$; $x_{24} \le 2200$; $x_{25} \le 2200$;

$x_{13} + x_{14} + x_{15} + x_{16} = 6500$; $x_{23} + x_{24} + x_{25} + x_{26} = 6000$;

$x_{13} + x_{23} = 5000$; $x_{14} + x_{24} = 4000$; $x_{15} + x_{25} = 3000$; $x_{16} + x_{26} = 500$;

all $x_{ij} \ge 0$.

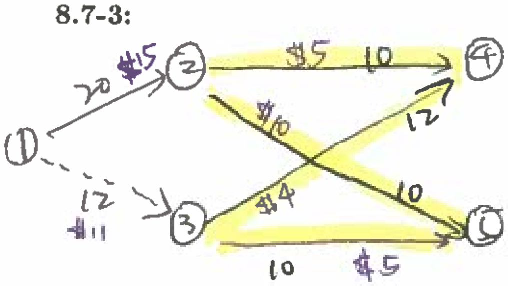
x_{13}	x_{14}	x_{15}	x_{16}	x_{23}	x_{24}	x_{25}	x_{26}		rhs	Constraint
1	1	1	1	0	0	0	0	=	6500	Node 1
0	0	0	0	1	1	1	1	=	6000	Node 2
-1	0	0	0	-1	0	0	0	=	-5000	Node 3
0	-1	0	0	0	-1	0	0	=	-4000	Node 4
0	0	-1	0	0	0	-1	0	=	-3000	Node 5
0	0	0	-1	0	0	0	-1	=	-500	Node 6
1	0	0	0	0	0	0	0	$\le$	2200	arc (1,3)
0	1	0	0	0	0	0	0	$\le$	2200	arc (1,4)
0	0	1	0	0	0	0	0	$\le$	2200	arc (1,5)
0	0	0	1	0	0	0	0	$\le$	2200	arc (1,6)
0	0	0	0	1	0	0	0	$\le$	2200	arc (2,3)
0	0	0	0	0	1	0	0	$\le$	2200	arc (2,4)
0	0	0	0	0	0	1	0	$\le$	2200	arc (2,5)
0	0	0	0	0	0	0	1	$\le$	2200	arc (2,6)

This problem is infeasible since City 1 demand 5000, but can at most receive $2200 * 2 = 4400$ units.

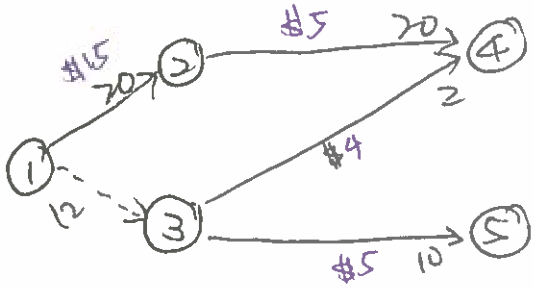
8.6-2:

Using the minimum spanning tree model, we connect (1,3), (3,5),(3,4), (3,2), yielding the total cost of 15.

8.7-3:



From the initial feasible solutions, we have: $y_1 = 0$; $y_2 = -15$; $y_3 = -16$; $y_4 = -20$; $y_5 = -35$.
From here, we got: $c_{54} = -35 + 20 - 14 = -29$; $c_{13} = 0 + 16 - 11 = 5$; $c_{23} = -15 + 16 - 5 = -4$; $c_{35} = -16 + 35 - 5 = 14$
c_{35} violates the optimal condition, so we find a cycle for arc (3,5), which is (3,5)-(5,2)-(2,4)-(4,3), and the quantity for arc (3,5) is 10. Adding this new arc, we got:



Now we check for the optimal condition again: $y_1 = 0$; $y_2 = -15$; $y_3 = -16$; $y_4 = -20$; $y_5 = -21$.
From here, we got: $c_{54} = -21 + 20 - 14 = -15$; $c_{13} = 0 + 16 - 11 = 5$; $c_{23} = -15 + 16 - 5 = -4$; $c_{25} = -10 - 15 + 21 = -4$. Since they all satisfy the optimal condition, we have reached the optimal solution: $x_{12} = 20$; $x_{24} = 20$; $x_{34} = 2$; $x_{35} = 10$; $x_{13} = 12$, and the total cost is 590.

8-1a:

Using Dijkstra's Algorithm:

[*NYC*	*CLE*	*St.L*	*Nash*	*Pho*	*Dal*	*SLC*	*LA*]
[0*	400*	950	800	*M*	*M*	*M*	*M*]
[0*	400*	950	800*	2200	1300	*M*	*M*]
[0*	400*	950*	800*	2200	1300	2000	*M*]
[0*	400*	950*	800*	2050	1300*	2000	*M*]
[0*	400*	950*	800*	2050	1300*	2000*	2600]
[0*	400*	950*	800*	2050*	1300*	2000*	2450]

The shortest path is NYC to St. Louis, to Phoenix, to LA, yielding 2450 gallons of fuel.

8-1b:

	CLE	St.L	Nash	Pho	Dal	SLC	LA	Supply
NYC	400	950	800	M	M	M	M	1
Cle	0	M	M	1800	900	M	M	1
St.L	M	0	M	1100	600	M	M	1
Nash	M	M	0	M	600	1200	M	1
Pho	M	M	M	0	M	M	400	1
Dal	M	M	M	900	0	1000	1300	1
SLC	M	M	M	M	M	0	600	1
Demand	1	1	1	1	1	1	1	

The optimal solution is:

	CLE	St.L	Nash	Pho	Dal	SLC	LA	Supply
NYC	400 0	950 1	800 0	M 0	M	M	M	1
Cle	0	M	M	1800 0	900	M	M	1
St.L	M	0	M	1100 1	600 0	M 0	M 0	1
Nash	M	M	0	M	600	1200	M 0	1
Pho	M	M	M	0	M	M	400 1	1
Dal	M	M	M	900	0	1000	1300 0	1
SLC	M	M	M	M	M	0	600 0	1
Demand	1	1	1	1	1	1	1	

Total usage of fuel: 2450.

8-1c:

min $Z=400x_{ny,clc} + 950x_{ny,StL} + 800x_{ny,nash} + 1800x_{clc,pho} + 900x_{clc,dal} + 1100x_{Stl,pho} + 600x_{StL,dal} +$

$600x_{nash,dal} + 1200x_{nash,SLC} + 400x_{pho,LA} + 900x_{dal,pho} + 1300x_{dal,LA} + 1000x_{dal,SLC} + 600x_{SLC,LA}$. Subject to:

$x_{ny,clc} + x_{ny,StL} + x_{ny,nash} = 1;$

$x_{ny,clc} = x_{clc,pho} + x_{clc,dal};$

$x_{ny,StL} = x_{StL,pho} + x_{StL,dal};$

$x_{ny,nash} = x_{nash,dal} + x_{nash,SLC};$

$x_{clc,pho} + x_{StL,pho} + x_{dal,pho} = x_{pho,LA};$

$x_{pho,LA} + x_{dal,LA} + x_{SLC,LA} = 1;$

$x_{nash,SLC} + x_{dal,SLC} = x_{SLC,LA};$

$x_{clc,dal} + x_{StL,dal} + x_{nash,dal} = x_{dal,pho} + x_{dal,LA} + x_{dal,SLC};$

With all $0 \leq x_{i,j} \leq 1$. The optimal solution is: $x_{ny,StL} = x_{StL,pho} = x_{pho,LA} = 1$, all other x's are 0. The minimum usage of fuel is 2450.