Sample Solution

Math 323 Operations Research HW11

9.2-1:
Max Z=$3x_1 + 2x_2 + 2x_3 + x_4 + 3x_5 + 3x_6 + x_7$, subject to:
$x_1 + x_3 + x_5 + x_7 \geq 4$
$x_3 + x_4 + x_5 + x_6 + x_7 \geq 2$
$x_2 + x_4 + x_6 \geq 1$
$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 5$
$3x_1 + 2x_2 + 2x_3 + x_4 + 3x_5 + 3x_6 + 3x_7 \geq 10$
$3x_1 + x_2 + 3x_3 + 3x_4 + 3x_5 + x_6 + 2x_7 \geq 10$
$x_1 + 3x_2 + 2x_3 + 3x_4 + 3x_5 + 2x_6 + 2x_7 \geq 10$
$x_6 + x_3 \leq 1$
$-x_4 - x_5 + 2 \leq 2y$
$x_1 \leq 2 - 2y$
$x_2 + x_3 \geq 1$
$x_1,\ x_2, ..., x_7,\ y$ are all 0 or 1 variables.
If $x_i = 1$, the player is a starter. Otherwise, he is not.

9.2-2:
Let $y_1,\ y_2,\ y_3$ represent site 1, 2, 3 respectively.
Min Z=$100000y_1 + 20x_1 + 60000y_2 + 30x_2 + 40000y_3 + 40x_3$, subject to:
$0.4x_1 + 0.25x_2 + 0.2x_3 \geq 80000$
$0.3x_1 + 0.2x_2 + 0.25x_3 \geq 50000$
$x_1 \leq My_1$
$x_2 \leq My_2$
$x_3 \leq My_3$
$y_1,\ y_2, y_3$ are all 0 or 1 variables. $x_1,\ x_2,\ x_3 \geq 0$.
If $y_i = 1$, the site is used to build stations. Otherwise, it is not. x_i represent the ton of water being treated under each site 1, 2, 3.

could specify the value of M.

$X_1 \leq 200000\, y_1$
$X_2 \leq 320000\, y_2$
$X_3 \leq 400000\, y_3$

9.2-3:
Let $y_1,\ y_2$ represent whether to produce product 1 and 2. Let x_1, x_2 represent the amount of product 1 and product 2 produced.
Max Z=$2x_1 + 5x_2 - 10y_1 - 20y_2$, subject to:
$3x_1 + 6x_2 \leq 120$,
$x_1 \leq 40y_1$,
$x_2 \leq 20y_2$.
$y_1,\ y_2$ are 0 or 1 variables. $x_1,\ x_2 \geq 0$.
If $y_j = 1$, then product i is produced. Otherwise, it is not.

9.2-4:
When $x_2 + x_3 = 2$, we want $x_4 = 1$.
We add constraint: $x_2 + x_3 - 1 \leq x_4$, with $x_2, x_3, x_4 = 0, 1$.

9.3-1:
Solving the IP relaxation problem as subproblem 1, we get:
$x_1 = 3.5$, $x_2 = 1.5$, $Z = 20.5$.
Now we divide the problem up based on x_1:
Subproblem 2: subproblem 1+$x_1 \leq 3$;
Subproblem 3: subproblem 1+$x_1 \geq 4$;
Solving subproblem 2, we get $x_1 = 3$, $x_2 = 2$, $Z = 19$,
And solving subproblem 3, we get $x_1 = 4$, $x_2 = 0$, $Z = 20$.

Since $20 > 19$ and both results yield integer solutions,
we have the optimal solution as $x_1 = 4, x_2 = 0, Z = 20$.

9.3-3:
Solving the IP relaxation problem as subproblem 1, we get:
$x_1 = 5$, $x_2 = 2.5$, $Z = 17.5$.
Now we divide the problem up based on x_2:
Subproblem 2: subproblem 1+$x_2 \leq 2$;
Subproblem 3: subproblem 1+$x_2 \geq 3$;
Solving subproblem 2, we get $x_1 = 17/3$, $x_2 = 2$, $Z = 52/3$,
And solving subproblem 3, we get $x_1 = 4$, $x_2 = 3$, $Z = 17$.
We further divide subproblem 2 based on x_1:
Subproblem 4: subproblem 2+$x_1 \leq 5$;
Subproblem 5: subproblem 2+$x_1 \geq 6$;
Solving subproblem 4, we get $x_1 = 5$, $x_2 = 2$, $Z = 16$,
And solving subproblem 5, we get $x_1 = 6$, $x_2 = 7/4$, $Z = 69/4$.
We further divide subproblem 5 based on x_2:
Subproblem 6: subproblem 5+$x_2 \leq 1$;
Subproblem 7: subproblem 5+$x_2 \geq 2$;
Solving subproblem 6, we get $x_1 = 7$, $x_2 = 1$, $Z = 17$,
And solving subproblem 7, we get $x_1 = 17/3$, $x_2 = 2$, $Z = 52/3$.
We further divide subproblem 7 based on x_1:
Subproblem 8: subproblem 7+$x_1 \leq 5$;
Subproblem 9: subproblem 7+$x_1 \geq 6$;
Solving subproblem 8, we get $x_1 = 5$, $x_2 = 2$, $Z = 16$,
And solving subproblem 9, we get an infeasible outcome.
Thus, we have finished all possibilities. Since $17 > 16$, we have the optimal solution as $x_1 = 4, x_2 = 3$, or $x_1 = 7, x_2 = 1, Z = 17$.