Homework 12

Sample Solution

Problem 9.5.2

min z= $60x_1 + 48x_2 + 14x_3 + 31x_4 + 10x_5$

subject to $800x_1 + 600x_2 + 300x_3 + 400x_4 + 200x_5 \leq 1100$

First node, we have $x_2 = x_4 = 1$ and $x_1 = \frac{1}{8}$. We branch at x_1, when $x_1 = 0$, we obtain node 2, where $x_1 = 0, x_2 = x_4 = 1$, $x_5 = 0.5$. When $x_1 = 1$, we obtain node 3, where $x_2 = x_1 = 1$, which will be infeasible. Thus, node 3 is terminated. Then, we branch node 2, we obtain node 4 when $x_1, x_5 = 0, x_2 = x_4 = 1$, $x_3 = \frac{1}{3}$. And we obtain node 5 when $x_5 = x_2 = 1$, $x_1 = 0$, and $x_4 = \frac{3}{4}$. From node 4, we obtain two other node. Node 6 is $x_1, x_3, x_5 = 0$, $x_2 = x_4 = 1$. Z=79. Node 7, $x_2 = x_3 = 1$, $x_1 = x_5 = 0$, $x_4 = \frac{1}{2}$, which will eventually be infeasible. From node 5, we obtain node 8,9. Node 8 is $x_3 = x_2 = x_5 = 1$ and $x_1 = x_4 = 0$ Z=70. and Node 9 returns $x_4 = x_5 = 1$, $x_2 = \frac{5}{6}$, $x_1 = x_3 = 0$. Node 9 create branch 10, $x_4 = x_5 = x_3 = 1$ and $x_1 = x_2 = 0$. Node 11 we obtain $x_2 = x_4 = x_5 = 1$ and $x_1 = x_3 = 0$, which is infeasible.

Thus, the optimal solution is reached when $x_2 = x_4 = 1$, $x_1 = x_3 = x_5 = 0$, Z=79.

10/10

9.6-1:

Total work time: 7+5+9+11=32

$x_1 : 32 - 14 = 18$

$x_2 : 32 - 13 = 19$

$x_3 : 32 - 18 = 14$

$x_4 : 32 - 15 = 17$

Thus, we choose x_3 to be the last.

Now the remaining work time is: 7+5+11=23

$x_1 : 23 - 14 = 9$

$x_2 : 23 - 13 = 10$

$x_4 : 23 - 15 = 8$

Thus, we choose x_4 to be the 3rd.

Now the remaining work time is: 7+5=12

$x_1 : 12 - 14 = -2$

$x_2 : 12 - 13 = -1$

We arbitrarily choose to finish x_2 first and then x_1. This arrangement yield a total delay time of 22.

Now we try to finish x_4 last: Now the remaining work time is: 7+5+9=21

$x_1 : 21 - 14 = 7$

$x_2 : 21 - 13 = 8$

$x_3 : 21 - 18 = 3$

Thus, we choose x_3 to be the 3rd.

$x_1 : 12 - 14 = -2$

$x_2 : 12 - 13 = -1$

We arbitrarily choose to finish x_2 first and then x_1. This arrangement yield a total delay time of 20. Checking the other two possibilities, we know that any arrangement that yields a delay time larger than 20 is not optimal. Thus, we conclude that this is the optimal solution: job 2 $\rightarrow job1 \rightarrow job3 \rightarrow job4$.

10/10

9.6-5:

NNH: $LFR \to LFP \to LP \to LR \to LFR$, which yields a total cost of 330.

CIH: Begin with LFR-LFP-LFR, we first consider the possibility of changing LFR-LFP: LFR-LR LFP which add 200 or LFR-LP-LFP which add 210.
Then we consider changing LFP-LFR. There are 2 possibilities: LFP-LR-LFR which add 170; and LFP-LP-LFR which add 180. Thus, we choose LFR-LFP-LR-LFR.
Now we consider where to add LP. There are three possibilities:
LFR-LP-LFP-LR-LFR which add 210 cost;
LFR-LFP-LP-LR-LFR which add 50 cost;
LFR-LFP-LR-LP-LFR which add 100 cost.
Thus, the optimal solution based on CIH is LFR-LFP-LP-LR-LFR with a total cost of 50+110+80+90=330.

10
10

9.7-1:

The final optimal answer is $x_1 = x_2 = x_4 = x_5 = 1, x_3 = 0, Z = 4$.

10