11 Consider the following LP:

$$\max z = 10x_1 + x_2$$
$$\text{s.t.} \quad x_1 \leq 1$$
$$20x_1 + x_2 \leq 100$$
$$x_1, x_2 \geq 0$$

a Find all the basic feasible solutions for this LP.

b Show that when the simplex is used to solve this LP, every basic feasible solution must be examined before the optimal solution is found.

By generalizing this example, Klee and Minty (1972) constructed (for $n = 2, 3, \ldots$) an LP with n decision variables and n constraints for which the simplex algorithm examines $2^n - 1$ basic feasible solutions before the optimal solution is found. Thus, there exists an LP with 10 variables and 10 constraints for which the simplex requires $2^{10} - 1 = 1{,}023$ pivots to find the optimal solution. Fortunately, such "pathological" LPs rarely occur in practical applications.

12 Productco produces three products. Each product requires labor, lumber, and paint. The resource requirements, unit price, and variable cost (exclusive of raw materials) for each product are given in Table 70. Currently, 900 labor hours, 1,550 gallons of paint, and 1,600 board feet of lumber are available. Additional labor can be purchased at $6 per hour, additional paint at $2 per gallon, and additional lumber at $3 per board foot. For the following two sets of priorities, use preemptive goal programming to determine an optimal production schedule. For set 1:

Priority 1 Obtain profit of at least $10,500.
Priority 2 Purchase no additional labor.
Priority 3 Purchase no additional paint.
Priority 4 Purchase no additional lumber.

For set 2:

Priority 1 Purchase no additional labor.
Priority 2 Obtain profit of at least $10,500.
Priority 3 Purchase no additional paint.
Priority 4 Purchase no additional lumber.

13 Jobs at Indiana University are rated on three factors:

Factor 1 Complexity of duties
Factor 2 Education required
Factor 3 Mental and or visual demands

For each job at IU, the requirement for each factor has been rated on a scale of 1–4, with a 4 in factor 1 representing high complexity of duty, a 4 in factor 2 representing high educational requirement, and a 4 in factor 3 representing high mental and/or visual demands.

TABLE 70

Product	Labor	Lumber	Paint	Price ($)	Variable Cost ($)
1	1.5	2	3	26	10
2	3	3	2	28	6
3	2	4	2	31	7