

Min cost:

15	8	6	10	9	35
30	9	12	13	7	50
0	14	9	16	5	40
	20		30	30	
	0			0	
					0

Proceed to do iteration to improve the optimal.

Vogel's method

8	6	10	9	35
9	12	13	7	50
14	9	16	5	40
	20		30	
9-8=1	9-6=3	13-10=3	7-5=2	
45	20	30	30	
	10			

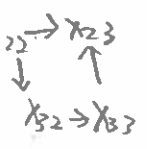
Reduced problem

$$\begin{array}{l}
 8-6=2 \\
 9-7=2 \\
 9-5=4 \\
 8-6=2 \\
 12-9=3 \\
 \leftarrow 14-9=5
 \end{array}$$

Example Solve the Powerco problem.

	C1	C2	C3	C4	Supply
P1	\$8	\$6	\$10	\$9	≤ 35
P2	\$9	\$12	\$13	\$7	≤ 50
P3	\$14	\$9	\$16	\$5	≤ 40
Demand	≥ 45	≥ 20	≥ 30	≥ 30	

Mind the typo of last lecture

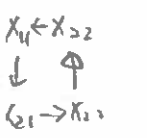


u_i	v_j	8	11	12	1	
		City1	City2	City3	City4	Supply
0		0/8 (35)	5/6	2/10	-8/9	35
1		0/9 (10)	0/12 (20-Δ)	0/13 (20+Δ)	-5/7	50
4		-2/14	0/9 (+Δ)	0/16 (10-Δ)	0/15 (30)	40
		45	20	30	30	

$u_1 =$ $v_1 =$
 $u_2 =$ $v_2 =$
 $u_3 =$ $v_3 =$

$\Delta = 10$
 ~~$\Delta = 20$~~

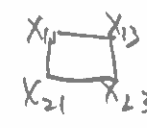
Better loop
 $x_{22} x_{23} x_{33} x_{32}$



u_i	v_j	8	11	12	7	
		City1	City2	City3	City4	Supply
0		0/8 (35)	5/6 (Δ)	2/10 (36)	-2/9	35
1		0/9 (10)	0/12 (10-Δ)	0/13 (36)	1/7	50
-2		-8/14	0/9 (10)	-6/16 (30)	0/15	40
		45	20	30	30	

$u_1 =$ $v_1 =$
 $u_2 =$ $v_2 =$
 $u_3 =$ $v_3 =$
 $u_4 =$ $v_4 =$

$\Delta = 10$



u_i	v_j	8	6	12	2	
		City1	City2	City3	City4	Supply
0		0/8 (25)	0/6 (10)	2/10 (+Δ)	-7/9	35
1		0/9 (20)	-5/12	0/13 (30)	-4/7	50
3		-5/14	0/9 (10)	-1/16 (30)	0/5	40
		45	20	30	30	

$u_1 =$ $v_1 =$
 $u_2 =$ $v_2 =$
 $u_3 =$ $v_3 =$
 $u_4 =$ $v_4 =$

$\Delta = 25$

	City1	City2	City3	City4	Supply
Plant1	8	6	10	9	35
Plant2	9	12	13	7	50
Plant3	14	9	16	5	40
Demand	45	20	30	30	

$u_1 =$ $v_1 =$
 $u_2 =$ $v_2 =$
 $u_3 =$ $v_3 =$
 $u_4 =$ $v_4 =$

$\Delta =$

All $u_i + v_j - c_{ij} \leq 0$. \therefore optimal

7.4 Sensitivity Analysis

- Change c_{ij} to $c_{ij} + \Delta$ for a nonbasic variable. Then use the same u_i, v_j and compute the new $c_{ij} - u_i - v_j$ to decide the range for Δ so that the solution to remain optimal. (For minimum cost, we want $c_{ij} - u_i - v_j \geq 0$.)
- Change c_{ij} to $c_{ij} + \Delta$ for a basic variable. We need to solve for u_i, v_j again to compute c_{rs} for all c_{rs} , and determine the range for the Δ so that the solution to remain optimal.
- Suppose u_i, v_j are changed by Δ .

If x_{ij} is a basic variable, change x_{ij} to $x_{ij} + \Delta$ and compute $Z = \sum_{r,s} c_{rs} x_{rs} c_{ij} \Delta$.

If x_{ij} is a nonbasic variable, find a loop starting with x_{ij} together with other basic variable. Then increase the odd vertex of the loop by Δ , and decrease the even vertex of the loop by Δ .

See Examples in p. 390-392.

TABLE 39
Optimal Tableau for Powerco

		City 1	City 2	City 3	City 4	Supply
		$v_1 = 4$	6	10	2	
Plant 1	$u_1 = 0$	8	10	25	9	35
Plant 2	3	45	12	5	7	50
Plant 3	3	14	10	16	30	40
Demand		45	20	30	30	

Solving these equations, we obtain $u_1 = 0$, $v_2 = 6$, $v_3 = 10 + \Delta$, $v_4 = 6 + \Delta$, $u_2 = 3 - \Delta$, $u_3 = 3$, and $v_5 = 2$.

We now price out each nonbasic variable. The current basis will remain optimal as long as each nonbasic variable has a nonpositive coefficient in row 0.

$$\bar{c}_{11} = u_1 + v_1 - 8 = \Delta - 2 \leq 0 \quad \text{for } \Delta \leq 2$$

$$\bar{c}_{14} = u_1 + v_4 - 9 = -7$$

$$\bar{c}_{22} = u_2 + v_2 - 12 = -3 - \Delta \leq 0 \quad \text{for } \Delta \geq -3$$

$$\bar{c}_{24} = u_2 + v_4 - 7 = -2 - \Delta \leq 0 \quad \text{for } \Delta \geq -2$$

$$\bar{c}_{31} = u_3 + v_1 - 14 = -5 + \Delta \leq 0 \quad \text{for } \Delta \leq 5$$

$$\bar{c}_{33} = u_3 + v_3 - 16 = \Delta - 3 \leq 0 \quad \text{for } \Delta \leq 3$$

Thus, the current basis remains optimal for $-2 \leq \Delta \leq 2$, or $8 = 10 - 2 \leq c_{13} \leq 10 + 2 = 12$.

TABLE 40
Optimal Tableau for Powerco II
 $s_1 = 35 + 2 = 37$ and
 $s_2 = 20 + 2 = 22$

		City 1	City 2	City 3	City 4	Supply
		$v_1 = 8$	10	10	2	
Plant 1	$u_1 = 0$	8	12	25	9	37
Plant 2	3	45	12	5	7	50
Plant 3	3	14	10	16	30	40
Demand		45	22	30	30	

TABLE 41
Optimal Tableau for Powerco II
 $s_1 = 35 + 1 = 36$ and
 $s_2 = 45 + 1 = 46$

		City 1	City 2	City 3	City 4	Supply
		$v_1 = 8$	8	10	2	
Plant 1	$u_1 = 0$	8	10	26	9	36
Plant 2	2	46	12	13	7	50
Plant 3	3	14	10	16	30	40
Demand		46	20	30	30	