Min cost:

				Supply
8 [15]	6 [20]	10 [0]	9 [0]	~~35~~ ~~15~~
9 [30]	12 [0]	13 [20]	7 [0]	~~50~~ ~~50~~ 20
14 [0]	9 [0]	16 [10]	5 [30]	~~40~~ 10
~~45~~ ~~30~~ 0	~~20~~ 0	30	~~30~~ 0	

Proceed to do iteration to improve the optimal.

Vogel's method

				Supply	
8	6	10	9	35	8-6=2
9	12	13	7	50	9-7=2
14	9 [10]	16	5 [30]	~~40~~	9-5=4
9-8=1	9-6=3	13-10=3	7-5=2		
45	~~20~~ 10	30	~~30~~		

Reduced problem

8-6 = 2

12-9 = 3

14-9 = 5 ←

Example Solve the Powerco problem.

	C1	C2	C3	C4	Supply
P1	\$8	\$6	\$10	\$9	≤ 35
P2	\$9	\$12	\$13	\$7	≤ 50
P3	\$14	\$9	\$16	\$5	≤ 40
Demand	≥ 45	≥ 20	≥ 30	≥ 30	

mind the typo of last lecture

v_j: 8, 11, 12, 1

u_i		City1	City2	City3	City4	Supply
0	Plant1	0 / 8 (35)	5 / 6	2 / 10	-8 / 9	35
1	Plant2	0 / 9 (10)	0 / 12 (20)−Δ	0 / 13 (20)+Δ	-5 / 7	50
4	Plant3	-2 / 14	Δ / 9 +Δ	0 / 16 (10)−Δ	0 / 5 (30)	40
	Demand	45	20	30	30	

$x_{22} \to x_{23}$, $x_{32} \to x_{33}$

$u_1 =$ $v_1 =$

$u_2 =$ $v_2 =$

$u_3 =$ $v_3 =$

$v_4 =$

$\Delta = 10$

~~$\Delta = 20$~~

Better loop $X_{22}\ X_{23}\ X_{33}\ X_{32}$

v_j: 8, 11, 12, 7

u_i		City1	City2	City3	City4	Supply
0	Plant1	0 / 8 (35)−Δ	5 / 6 +Δ	2 / 10	2 / 9	35
1	Plant2	0 / 9 (10)+Δ	0 / 12 (10)−Δ	0 / 13 (30)	1 / 7	50
-2	Plant3	-8 / 14	0 / 9 (10)	-6 / 16	0 / 5 (30)	40
	Demand	45	20	30	30	

$x_{11} \leftarrow x_{12}$, $x_{21} \to x_{22}$

$u_1 =$ $v_1 =$

$u_2 =$ $v_2 =$

$u_3 =$ $v_3 =$

$v_4 =$

$\Delta = 10$

v_j: 8, 6, 12, 2

u_i		City1	City2	City3	City4	Supply
0	Plant1	0 / 8 (25)−Δ	0 / 6 (10)	2 / 10 +Δ	-7 / 9	35
1	Plant2	0 / 9 (20)+Δ	-5 / 12	0 / 13 (30)−Δ	-4 / 7	50
3	Plant3	-5 / 14	0 / 9 (10)	-1 / 16	0 / 5 (30)	40
	Demand	45	20	30	30	

X_{11} — X_{13}, X_{21} — X_{23}

$u_1 =$ $v_1 =$

$u_2 =$ $v_2 =$

$u_3 =$ $v_3 =$

$v_4 =$

$\Delta = 25$

	City1	City2	City3	City4	Supply
Plant1	8	6 (10)	10 (25)	9	35
Plant2	9 (45)	12	13 (5)	7	50
Plant3	14	9 (10)	16	5 (30)	40
Demand	45	20	30	30	

$u_1 =$ $v_1 =$

$u_2 =$ $v_2 =$

$u_3 =$ $v_3 =$

$v_4 =$

$\Delta =$

All $u_i + v_j - c_{ij} \le 0$. $\therefore$ optimal

7.4 Sensitivity Analysis

- Change c_{ij} to $c_{ij} + \Delta$ for a nonbasic variable. Then use the same u_i, v_j and compute the new $c_{ij} - u_i - v_j$ to decide the range for Δ so that the solution to remain optimal. (For minimum cost, we want $c_{ij} - u_i - v_j \geq 0$.)

- Change c_{ij} to $c_{ij} + \Delta$ for a basic variable. We need to solve for u_i, v_j again to compute c_{rs} for all c_{rs}, and determine the range for the Δ so that the solution to remain optimal.

- Suppose u_i, v_j are changed by Δ.

  If x_{ij} is a basic variable, change x_{ij} to $x_{ij} + \Delta$ and compute $Z = \sum_{r,s} c_{rs} x_{rs} c_{ij} \Delta$.

  If x_{ij} is a nonbasic variable, find a loop starting with x_{ij} together with other basic variable. Then increase the odd vertex of the loop by Δ, and decrease the even vertex of the loop by Δ.

See Examples in p. 390-392.

TABLE 39
Optimal Tableau for Powerco

		City 1	City 2	City 3	City 4	Supply
	$v_j =$	6	6	10	2	
Plant 1	$u_1 = 0$	[8]	[6] 10	[10] 25	[9]	35
Plant 2	3	[9] 45	[12]	[13] 5	[7]	50
Plant 3	3	[14]	[9] 10	[16]	[5] 30	40
Demand		45	20	30	30	

Solving these equations, we obtain $u_1 = 0$, $v_2 = 6$, $v_3 = 10 + \Delta$, $v_1 = 6 + \Delta$, $u_2 = 3 - \Delta$, $u_3 = 3$, and $v_4 = 2$.

We now price out each nonbasic variable. The current basis will remain optimal as long as each nonbasic variable has a nonpositive coefficient in row 0.

$$\bar{c}_{11} = u_1 + v_1 - 8 = \Delta - 2 \le 0 \qquad \text{for } \Delta \le 2$$
$$\bar{c}_{14} = u_1 + v_4 - 9 = -7$$
$$\bar{c}_{22} = u_2 + v_2 - 12 = -3 - \Delta \le 0 \qquad \text{for } \Delta \ge -3$$
$$\bar{c}_{24} = u_2 + v_4 - 7 = -2 - \Delta \le 0 \qquad \text{for } \Delta \ge -2$$
$$\bar{c}_{31} = u_3 + v_1 - 14 = -5 + \Delta \le 0 \qquad \text{for } \Delta \le 5$$
$$\bar{c}_{33} = u_3 + v_3 - 16 = \Delta - 3 \le 0 \qquad \text{for } \Delta \le 3$$

Thus, the current basis remains optimal for $-2 \le \Delta \le 2$, or $8 = 10 - 2 \le c_{13} \le 10 + 2 = 12$.

TABLE 40
Optimal Tableau for Powerco If $s_1 = 35 + 2 = 37$ and $d_2 = 20 + 2 = 22$

		City 1	City 2	City 3	City 4	Supply
	$v_j =$	6	6	10	2	
Plant 1	$u_1 = 0$	[8]	[6] 12	[10] 25	[9]	37
Plant 2	3	[9] 45	[12]	[13] 5	[7]	50
Plant 3	3	[14]	[9] 10	[16]	[5] 30	40
Demand		45	22	30	30	

TABLE 41
Optimal Tableau for Powerco II $s_1 = 35 + 1 = 36$ and $d_1 = 45 + 1 = 46$

		City 1	City 2	City 3	City 4	Supply
	$v_j =$	6	6	10	2	
Plant 1	$u_1 = 0$	[8]	[6] 10	[10] 26	[9]	36
Plant 2	3	[9] 46	[12]	[13] 4	[7]	50
Plant 3	3	[14]	[9] 10	[16]	[5] 30	40
Demand		46	20	30	30	