**Math 323** QUIZ 3 Name: Sample Solution

**1.** There are 3 reservoirs R1, R2, R3 with daily supplies of 15, 20, and 25 million liters of water, respectively. On each day we must supply for cities C1, C2, C3, C4, whose deands are 8, 10, 12, 15 million liters, respectively. The cost of pumping per million liters is given by:

| | C1 | C2 | C3 | C4 |
|---|---|---|---|---|
| R1 | 2 | 3 | 4 | 5 |
| R2 | 3 | 2 | 5 | 2 |
| R3 | 4 | 1 | 2 | 3 |

(a) Set up the table for a balanced transportation problem with the given data.
(You should get a $3 \times 5$ table with decision variables $x_{ij}$.)

| | | | | | Supplies |
|---|---|---|---|---|---|
| $\boxed{2}$ | $\boxed{3}$ | $\boxed{4}$ | $\boxed{5}$ | $\boxed{0}$ | 15 |
| $\boxed{3}$ | $\boxed{2}$ | $\boxed{5}$ | $\boxed{2}$ | $\boxed{0}$ | 20 |
| $\boxed{4}$ | $\boxed{1}$ | $\boxed{2}$ | $\boxed{3}$ | $\boxed{0}$ | 25 |
| Demands: 8 | 10 | 12 | 15 | 15 | |

(b) Use NW corner rule to find an initial basic feasible solution.

| | | | | |
|---|---|---|---|---|
| 8 | 7 | | | |
| | 3 | 12 | 5 | |
| | | | 10 | 15 |

(c) Apply an iteration to the basic feasible solution:

$$(x_{11}, x_{12}, x_{15}, x_{22}, x_{24}, x_{33}, x_{35}) = (8, 5, 2, 5, 15, 12, 13).$$

| u\v | 2 | 3 | 2 | 3 | 0 | |
|---|---|---|---|---|---|---|
| 0 | 0 $\boxed{2}$ 8 | 0 $\boxed{3}$ 5 | -2 $\boxed{4}$ | -2 $\boxed{5}$ | 0 $\boxed{0}$ 2 | 15 |
| -1 | -2 $\boxed{3}$ | 0 $\boxed{2}$ 5 | -4 $\boxed{5}$ | 0 $\boxed{2}$ 15 | -1 $\boxed{0}$ | 20 |
| 0 | -2 $\boxed{4}$ | 2 $\boxed{1}$ | 0 $\boxed{2}$ 12 | 0 $\boxed{3}$ | 0 $\boxed{0}$ 13 | 25 |

loop

$X_{32} \to X_{35} \to X_{15} \to X_{12}$

$\Delta = 5$.

(d) Check whether the new solution is optimal.

| | 2 | 1 | 2 | 1 | 0 | |
|---|---|---|---|---|---|---|
| 0 | 0 $\boxed{2}$ 8 | -2 $\boxed{3}$ | -2 $\boxed{4}$ | -4 $\boxed{5}$ | 0 $\boxed{0}$ 7 | 15 |
| 1 | 0 $\boxed{3}$ | 0 $\boxed{2}$ 5 | -2 $\boxed{5}$ | 0 $\boxed{2}$ 15 | 1 $\boxed{0}$ | 20 |
| 0 | -2 $\boxed{4}$ | 0 $\boxed{1}$ 5 | 0 $\boxed{2}$ 12 | -2 $\boxed{3}$ | 0 $\boxed{0}$ 8 | 25 |
| | 8 | 10 | 12 | 15 | 15 | |

| | 2 | 1 | 2 | 2 | 0 | |
|---|---|---|---|---|---|---|
| 0 | 0 8 | -2 | -2 | -3 | 0 7 | 15 |
| → 0 | -1 | -1 | -3 | 0 15 | 0 5 | 20 |
| 0 | -2 | 0 10 | 0 12 | -2 | 0 3 | 25 |
| | 8 | 10 | 12 | 15 | 15 | |

$z = 16 + 10 + 30 + 5 + 24$

$z = 16 + 30 + 10 + 24$ (Optimal

### 7.5 Assignment problems

Assign $m$ people to $m$ jobs with a cost (salary) matrix. Basically, we are solving the supply demand problem with $s_i = v_j = 1$ for all $i, j = 1, \dots, m$.

We can use the standard transportation simplex method.

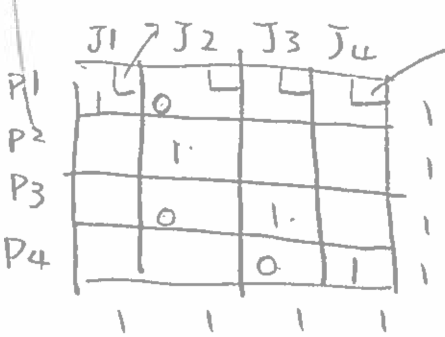
We can also use the Hungarian method.

1. Subtract the minimum element in each row from entries of the same row for all rows to get a reduced cost matrix.
2. Find the minimum number of lines (vertical / horizontal) covering all the zeros in the matrix. If we need $m$ lines, we are done. Else, go to Step 3.
3. Find the minimum entry not covered by the lines, and subtract it from the uncovered entries, and add it to entries covered by two lines. Return to Step 2.

Explanation of Algorithm. Adding/subtracting constants from a row/columns does not change the optimal solution.

Steps 1 and 3 are just changing the cost matrix without changing the optimal solution. In particular, Step 3 is adding a constant to the covered rows, and subtracting a the same constant from the uncovered columns.

**See Example in p. 396-397.**




effectiveness

$4+4-1=7$. $x_{ij}$ are basic variables.

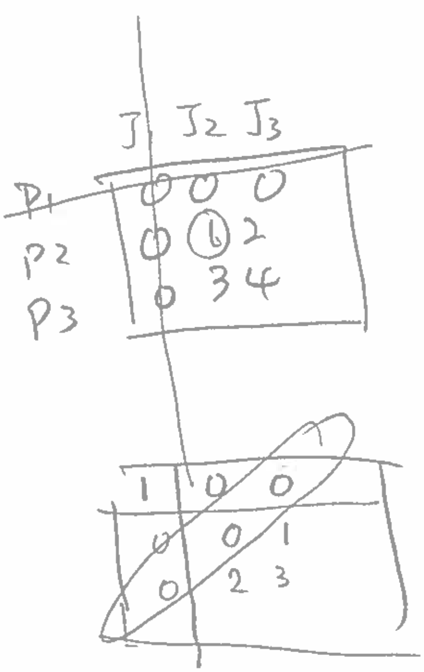


**TABLE 44**
Basic Feasible Solution for Machineco

| | $u_i$ | Job 1 | Job 2 | Job 3 | Job 4 | |
|---|---|---|---|---|---|---|
| $v_j =$ | | 3 | 4 | 8 | 7 | |
| Machine 1 | $u_1 = 0$ | [14] | [5] 1 | [8] 0 | [7] 0 | 1 |
| Machine 2 | −2 | [2] | [12] | [6] 2 | [5] 1 | 1 |
| Machine 3 | −5 | [7] | [8] | [3] 1 | [9] 2 | 1 |
| Machine 4 | −1 | [2] 1 | [4] 0 | [6] | [10] | 1 |
| | | 1 | 1 | 1 | 1 | |

**TABLE 45**
$x_{43}$ Has Entered the Basis

| | $u_i$ | Job 1 | Job 2 | Job 3 | Job 4 | |
|---|---|---|---|---|---|---|
| $v_j =$ | | 3 | 5 | 7 | 7 | |
| Machine 1 | $u_1 = 0$ | [14] | [5] 1 | [8] | [7] 0 | 1 |
| Machine 2 | −2 | [2] | [12] | [6] | [5] 1 | 1 |
| Machine 3 | −4 | [7] | [8] | [3] 1 | [9] | 1 |
| Machine 4 | −1 | [2] 1 | [4] 0 | [6] 0 | [10] | 1 |
| | | 1 | 1 | 1 | 1 | |

In the graphical representation

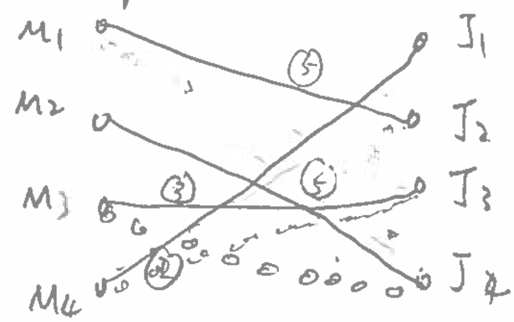


**TABLE 46**
Cost Matrix for Machineco

| | | | | Row Minimum |
|---|---|---|---|---|
| 14 | 5 | 8 | 7 | 5 |
| 2 | 12 | 6 | 5 | 2 |
| 7 | 8 | 3 | 9 | 3 |
| 2 | 4 | 6 | 10 | 2 |

**TABLE 47**
Cost Matrix After Row Minimums Are Subtracted

| | | | |
|---|---|---|---|
| 9 | 0 | 3 | 2 |
| 0 | 10 | 4 | 3 |
| 4 | 5 | 0 | 6 |
| 0 | 2 | 4 | 8 |
| Column Minimum | 0 | 0 | 2 |

**TABLE 48**
Cost Matrix After Column Minimums Are Subtracted

| | | | |
|---|---|---|---|
| [illegible] | 0 | 3 | 0 |
| 0 | 10 | 4 | 1 |
| [illegible] | 5 | 0 | 4 |
| 0 | 2 | 4 | 6 |

**TABLE 49**
Four Lines Required; Optimal Solution Is Available

| | | | |
|---|---|---|---|
| 10 | 0 | 3 | 0 |
| 0 | 9 | 3 | 0 |
| [illegible] | 5 | 0 | [illegible] |
| 0 | 1 | 3 | 5 |

Thus, we have found the optimal assignment $x_{12} = 1$, $x_{24} = 1$, $x_{33} = 1$, and $x_{41} = 1$. Of course, this agrees with the result obtained by the transportation simplex

### 7.6 Transshipment problems

**Basic idea** Using the intermediate nodes (transshipment points) in a transportation network as as supply and demand points as well.

**Example (pp. 400-402)** Widgetco manufactures widgets at Memphis and Denver (150 unit and 200 units). Have to ship to Los Angeles and Boston (130 unites in each demand point).

Looking at the transportation cost matrix, one would assume that Memphis, Denver, N.Y. Chicago as supply points, N.Y. Chicago, L.A., Boston, Dummy as demand points.

Solving the problem, we see that it is beneficial to send product from Memphis to L.A. through N.Y.

**TABLE 58**
Shipping Costs for Transshipments

| From | To ($) Memphis | Denver | N.Y. | Chicago | L.A. | Boston |
|---|---|---|---|---|---|---|
| Memphis | 0 | — | 8 | 13 | 25 | 28 |
| Denver | — | 0 | 15 | 12 | 26 | 25 |
| N.Y. | — | — | 0 | 6 | 16 | 17 |
| Chicago | — | — | 6 | 0 | 14 | 16 |
| L.A. | — | — | — | — | 0 | — |
| Boston | — | — | — | — | — | 0 |

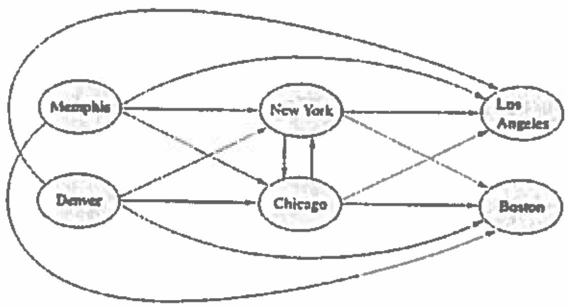



**FIGURE 9**
A Transshipment Problem

Memphis: 130 + 20 = 150
Denver: 130 + 70 = 200
N.Y.: 220 + 130 − 130 − 220 = 0
Chicago: 350 − 350 = 0
L.A.: −130
Boston: −130
Dummy: −20 − 70 = −90

**TABLE 59**
Representation of Transshipment Problem as Balanced Transportation Problem

| | N.Y. | Chicago | L.A. | Boston | Dummy | Supply |
|---|---|---|---|---|---|---|
| Memphis | 8; 130 | 13 | 25 | 28 | 0; 20 | 150 |
| Denver | 15 | 12 | 26 | 25; 130 | 0; 70 | 200 |
| N.Y. | 0; 220 | 6 | 16; 130 | 17 | 0 | 350 |
| Chicago | 6 | 0; 350 | 14 | 16 | 0 | 350 |
| Demand | 350 | 350 | 130 | 130 | 90 | |

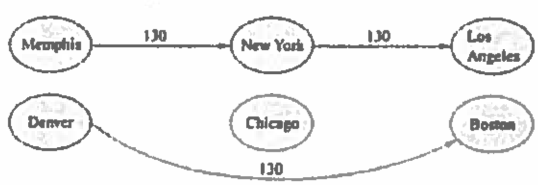



**FIGURE 10**
Optimal Solution to Widgetco